Queen Mary
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Complex Networks (MTH6142)
Formative Assignment 7

- **Growing network model**
  Consider the following model for a growing simple network.
  We adopt the following notation: N and L indicate respectively the total number of nodes and links of the network, A_{ir} indicates the generic element of the adjacency matrix $\mathbf{A}$ of the network and k_i indicates the degree of node i.
  At time $t = 0$ the network is formed by a $n_0 = 2$ nodes and a single link (initial number of links $m_0 = 1$) connecting the two nodes.
  At every time step $t > 0$ the network evolve according to the following rules:

  - A single new node joins the network.
  - An existing link (i, r) between a node i and a node r (two nodes of the network) is chosen randomly with uniform probability

    $$\pi_{(i,r)} = \frac{A_{ir}}{L}$$

    and the new node is linked to both node i and node r.

  a) Show that in this network evolution at each time step the average number of links $\tilde{\Pi}_i$ added to node i follows the preferential attachment rule, i.e.

  $$\tilde{\Pi}_i = \sum_{r=1}^{N} \pi_{(i,r)} = 2\frac{k_i}{\sum_{j=1}^{N} k_j}.$$

  b) What is the total number of links in the network at time t? What is the total number of nodes?

  c) What is the average degree $\langle k \rangle$ of the network at time t?

  d) Use the result at point a) to derive the time evolution $k_i = k_i(t)$ of the average degree k_i of a node i for $t \gg 1$ in the mean-field, continuous approximation.

  e) What is the degree distribution of the network at large times in the mean-field approximation?

f) Let $N_k(t)$ be the average number of nodes with degree k at time t. Write the master equation for $N_k(t)$.

g) Solve the master equation, finding the exact result for the degree distribution $P(k)$ in the limit $N \to \infty$.