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A measure of centrality based on network efficiency

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Abstract. We introduce *delta centralities*, a new class of measures of structural centrality for networks. In particular, we focus on a measure in this class, the information centrality C^I, which is based on the concept of efficient propagation of information over the network. C^I is defined for both valued and non-valued graphs, and applies to groups as well as individuals. The measure is illustrated and compared with respect to the standard centrality measures by using a classic network data set. The statistical distribution of information centrality is investigated by considering large computer generated graphs and two networks from the real world.

Contents

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1. Introduction

The idea of centrality was first applied to human communication by Bavelas [1] who was interested in the characterization of the communication in small groups of people and assumed a relation between *structural centrality* and influence in group processes. Since then, various measures of centrality have been proposed over the years to quantify the importance of an individual in a social network [2]. More recently, the issue of structural centrality has attracted the attention of physicists [3]–[5], who have extended its applications to the realm of biological and technological networks (see for instance [6]–[8]). The standard centrality measures can be divided into two classes: those based on the idea that the centrality of a node in a network is related to how it is *near* to other nodes, and those based on the idea that central nodes stand *between* others, playing the role of intermediary [2]. Degree [9] and closeness [10] are examples of measures of the first kind, while shortest-path [11] or flow [12] betweenness are measures of the second kind. In this paper, we propose a new class of centrality measures, the so-called *delta centrality* (or Δ *centralities*), which is a combination of the two main classes of centrality mentioned above. Delta centralities measure the contribution of a node to a network cohesiveness property, from the observed variation in such property (hereafter called 'delta centrality') when the node is deleted. In particular, we study one measure of this class, the *information centrality*, based on the concept of efficient propagation of information over the network [13, 14]. The information centrality of a node is defined as the relative drop in the network efficiency caused by the removal of the node from the network. In other words, the information centrality measures how communication over the network is affected by the deactivation of the node. The information centrality is defined for both valued and non-valued graphs, and even more importantly, it naturally applies to groups of nodes as well as single nodes.

The paper is organized as follows. In section 2, we present a brief review of the standard measures. In section 3, we introduce the delta centralities and the information centrality; we then illustrate similarities and dissimilarities with respect to the standard measures by means of a few simple examples. In section 4, we draw our conclusions.

2. Classic measures of centrality

Centrality measures will be illustrated and framed in their natural historical context, that of social networks. A social network is here represented as a undirected, non-valued graph $\mathbf{G}$, consisting of a set of N nodes (or vertices) and a set of K edges (or lines) connecting pairs of nodes. The nodes of the graph are the individuals, the actors of a social group, and the lines represent the social links. We describe the graph by the so-called adjacency matrix, a $N \times N$ matrix whose entry a_{ij} is 1 if there is an edge between i and j and 0 otherwise. The entries on the diagonal, values of a_{ii}, are undefined and for convenience are set equal to 0.

The *degree centrality* is based on the idea that important nodes are those with the largest number of ties to other nodes in the graph. The degree centrality of a node i is defined as [9]:

$$C_i^{\mathrm{D}} = \frac{k_i}{N-1} = \frac{\sum_{j \in \mathbf{G}} a_{ij}}{N-1}, \tag{1}$$

where k_i is the degree of node i.

The *closeness centrality* of a node i is based on the concept of minimum distance or geodesic d_{ij}, i.e. the minimum number of edges traversed to get from i to j [5] and is defined as [2, 10]:

$$C_i^{\mathrm{C}} = (L_i)^{-1} = \frac{N-1}{\sum_{j\in\mathbf{G}} d_{ij}}, \tag{2}$$

where L_i is the average distance from i to all the other nodes.

The *betweenness centrality*, in its basic version, is defined by assuming that the communication travels just along the geodesic. If n_{jk} is the number of geodesics linking two nodes j and k, and $n_{jk}(i)$ is the number of geodesics linking the two nodes j and k that contain node i, the betweenness centrality of node i can be defined as [11]:

$$C_i^{\mathrm{B}} = \frac{1}{(N-1)(N-2)} \sum_{j\in\mathbf{G}, j\neq i} \sum_{k\in\mathbf{G}, k\neq i, k\neq j} n_{jk}(i)/n_{jk}. \tag{3}$$

There are several extensions for cases in which communication does not travel through geodesic paths only [12, 15, 16]. In particular, the flow betweenness is defined by assuming that each edge of the graph is like a pipe and can carry a unitary amount of flow [12]. By considering a generic node j as the source of flow, and a generic node k as the target, it is possible to calculate the maximum possible flow from j to k by means of the min-cut, max-flow theorem [17, 18]. The *flow betweenness centrality* of node i is defined as:

$$C_i^{\mathrm{F}} = \frac{\sum_{j,k\in\mathbf{G}} m_{jk}(i)}{\sum_{j,k\in\mathbf{G}} m_{jk}}, \tag{4}$$

where $m_{jk}(i)$ is the amount of flow passing through i when the maximum flow m_{jk} is exchanged from j to k.

All the centrality measures defined above can also be extended to quantify the importance of a group of nodes. Nevertheless such an extension (and the relative normalization) is not unique and a series of conventions must be adopted [19]. Conversely, the centrality measures we are going to introduce in the next section are defined in a natural way both for individuals and groups of individuals.

3. Delta centralities: the information centrality

Delta centrality measures are based on the following idea: the importance of a node (group of nodes) is related to the ability of the network to respond to the deactivation of the node (group of nodes) from the network. If $\mathbf{G}$ is the graph representing the network, the *delta centrality* (or Δ *centrality*), of node i, C_i^{Δ}, is defined as:

$$C_i^{\Delta} = \frac{(\Delta P)_i}{P} = \frac{P[\mathbf{G}] - P[\mathbf{G}']}{P[\mathbf{G}]}, \tag{5}$$

where P is a generic quantity measuring the cohesiveness of the graph, and $(\Delta P)_i$ is the variation of P under the deactivation (isolation) of node i, i.e. the removal from the graph of the edges incident in node i. By $\mathbf{G}'$ we indicate the graph obtained by removing from $\mathbf{G}$ the edges incident in node i. The precise definition of P in formula (5) does not need to be better specified. There are only some general restrictions on the choice of the quantity P. For instance, we require $(\Delta P)_i \geqslant 0$ for any node i of the graph. Of course, the meaning and effectiveness of the centrality measure C^{Δ} will depend on the choice of P. The simplest possibility is to take $P[\mathbf{G}] = K$, where K is the number of edges in the graph $\mathbf{G}$. In this case, $(\Delta P)_i$ will be proportional to the degree and the degree centrality, C_i^{D}, of i. A more interesting example is to take as P the *efficiency* E of the graph, a quantity introduced in [13] and measuring how well the nodes of the graph exchange information. This quantity is based on the assumption that the information/communication in a network travels along the shortest routes, and that the efficiency in the communication between two nodes i and j is equal to $1/d_{ij}$. The efficiency $E[\mathbf{G}]$ of a graph $\mathbf{G}$ is defined as:

$$E[\mathbf{G}] = \frac{1}{N(N-1)} \sum_{i \neq j \in \mathbf{G}} \frac{1}{d_{ij}}, \tag{6}$$

and measures the mean flow-rate of information over $\mathbf{G}$ [13, 14]. In such case, the delta centrality of a node i (of a group of nodes S), that we name *information centrality* C_i^{I} (C_S^{I}), is defined as the relative drop in the network efficiency caused by the deactivation of i (of S) from the graph $\mathbf{G}$:

$$C_i^{\mathrm{I}} = \frac{\Delta E}{E} = \frac{E[\mathbf{G}] - E[\mathbf{G}']}{E[\mathbf{G}]}. \tag{7}$$

In this formula we indicate by $\mathbf{G}'$ the graph of N nodes obtained by removing from $\mathbf{G}$ the edges incident in node i (in nodes belonging to S)[4]. (A similar idea can also be applied to define the importance of the edges of a graph [20]–[22].) The removal of some of the edges affects the communication between various nodes of the graph increasing the length of the shortest paths, consequently $E[\mathbf{G}'] \leqslant E[\mathbf{G}]$. The measure C_i^{I}, as the standard measures, is normalized to take values in the interval [0, 1]. It is immediately seen that C^{I} is somehow correlated to the three standard measures C^{D}, C^{C} and C^{B}. In fact the information centrality of node i depends on k_i, since the efficiency $E[\mathbf{G}']$ is smaller if the number of edges removed from the original graph is larger. C_i^{I} is correlated to C_i^{C} since the efficiency of a graph is connected to $(\sum_i L_i)^{-1}$. Finally C_i^{I}, similarly to C_i^{B}, depends on the number of geodesics passing by i, but it also depends on the lengths of the new geodesics, the alternative paths that are used as communication channels, once the node i is deactivated. No information about the new shortest paths is contained in C_i^{B}, and in the other two standard measures.

3.1. Simple examples

The information centrality agrees with the standard measures on assignment of extremes: for instance it gives the maximum importance to the central node of a star, and equal importance to

[4] A valued graph is a better description of a social system if the intensity of the social relations is a relevant ingredient that one wants to take into account. The same definition applies to valued graphs with the only difference that weighted shortest paths should be used in the definition of the efficiency [13, 14].

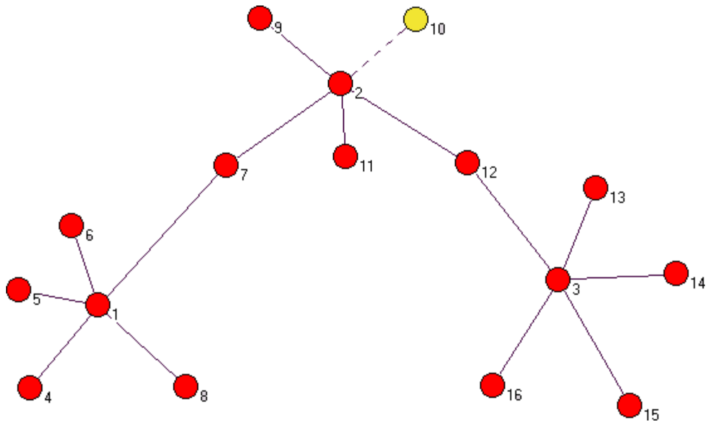

Figure 1. The tree $\mathbf{T_1}$ with $N = 16$ nodes reported in figure, and the tree $\mathbf{T_2}$, obtained from $\mathbf{T_1}$ by disconnecting node 10 from the rest of the graph, are used to compare the centrality measures.

the nodes of a complete graph. However the agreement breaks down between these extremes. Consider, for instance, a graph $\mathbf{G}$ composed by two main parts, graph $\mathbf{G_1}$ with N_1 nodes and graph $\mathbf{G_2}$ with N_2 nodes ($N_1 > N_2$), and by a single node n, connecting $\mathbf{G_1}$ to $\mathbf{G_2}$. For such a simple example C^{I} assigns, similarly to C^{B}, the maximum importance to node n, which certainly plays an important role since it works as a bridge between $\mathbf{G_1}$ and $\mathbf{G_2}$. On the other hand it is very unlikely that C^{D} or C^{C} would attribute the highest score to n.[5]

We now show that, in graphs with no cycles (trees), as $\mathbf{T_1}$ and $\mathbf{T_2}$ in figure 1, C^{I} is more similar to C^{C} than to C^{D} and C^{B} concerning resolution and stability. The four centrality scores for $\mathbf{T_1}$ are shown in table 1 left, where nodes are ordered in decreasing order of C^{I}. Although all the four measures attribute the highest centrality to node 2, there are some differences worth mentioning. As better shown in table 1 top-right, C^{I} assigns the top score in $\mathbf{T_1}$ to node 2, second score to nodes 1, 3, third score to nodes 7, 12, and is also able to disentangle nodes 9, 10, 11 (fourth score) from the remaining ones. The only other measure that operates such a distinction is C^{C} which, on the other hand, assigns the second score to nodes 7, 12 and the third score to nodes 1, 3 inverting the result of C^{I}. Neither C^{D} nor C^{B} have the resolution of C^{I} and C^{C}. In fact C^{D} assigns the top score to the three nodes 1, 2, 3, all having five neighbours, and the second score to nodes 7, 12 both with two neighbours. While C^{B} assigns the top score to node 2 and the second score to nodes 1, 3, 7, 12. Both C^{D} and C^{B} do not distinguish nodes 9, 10, 11 from the remaining ones. The node ranking obtained in $\mathbf{T_2}$ is reported in table 1 bottom-right. While in tree $\mathbf{T_1}$ nodes 2, 1 and 3 have the same number of neighbours, in $\mathbf{T_2}$, node 2 has less neighbours than nodes 1 and 3: this affects the node rankings based on C^{D} and C^{B}, while it does not change the rankings based on C^{I} and C^{C}.

3.2. Individuals and groups

As an example of a graph with cycles, we now study a real database recording 3 months of interactions amongst a group of 20 monkeys, where interactions were defined as joint presence

[5] In fact, $\mathbf{G_1}$ and $\mathbf{G_2}$ may contain nodes with degree larger than k_n. Moreover the node with smallest distance to all the other nodes will probably be in $\mathbf{G_1}$, especially if we assume $N_1 \gg N_2$.

Table 1. Left: centrality values for the nodes of $\mathbf{T_1}$. Right: centrality rankings in $\mathbf{T_1}$ (top) and $\mathbf{T_2}$ (bottom).

Node	C^I	C^D	C^C	C^B
2	0.591	0.333	0.455	0.714
1	0.444	0.333	0.349	0.476
3	0.444	0.333	0.349	0.476
7	0.389	0.133	0.405	0.476
12	0.389	0.133	0.405	0.476
9	0.116	0.067	0.319	0.000
10	0.116	0.067	0.319	0.000
11	0.116	0.067	0.319	0.000
4	0.106	0.067	0.263	0.000
5	0.106	0.067	0.263	0.000
6	0.106	0.067	0.263	0.000
8	0.106	0.067	0.263	0.000
13	0.106	0.067	0.263	0.000
14	0.106	0.067	0.263	0.000
15	0.106	0.067	0.263	0.000
16	0.106	0.067	0.263	0.000

Rank	C^I	C^D	C^C	C^B
1	2	1, 2, 3	2	2
2	1, 3	7, 12	7, 12	1, 3, 7, 12
3	7, 12	Others	1, 3	Others
4	9, 10, 11		9, 10, 11	
5	Others		Others	

Rank	C^I	C^D	C^C	C^B
1	2	1, 3	2	2
2	1, 3	2	7, 12	1, 3
3	7, 12	7, 12	1, 3	7, 12
4	9, 11	Others	9, 11	Others
5	Others		Others	

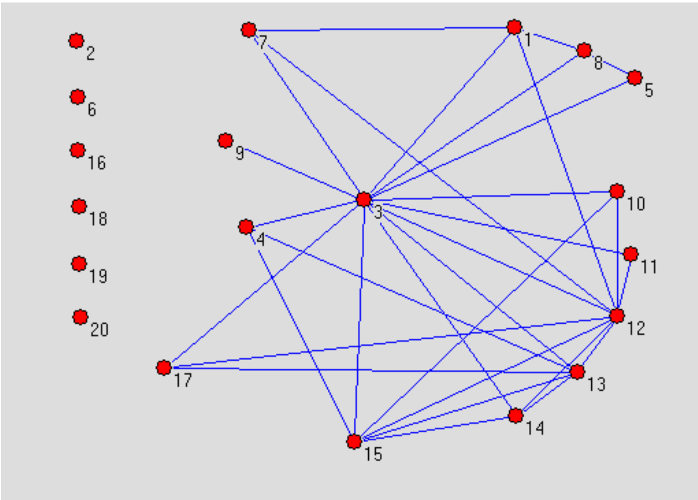

Figure 2. The graph of the interactions amongst a group of 20 monkeys [19]. The data set contains also information on the sex and age of each animal (see table 2).

at the river [19]. The resulting graph, shown in figure 2, consists in 6 isolated nodes and a connected component of 14 nodes. Such a graph has been studied in [19], where the standard node centrality measures C^D, C^C and C^D have been generalized to apply to groups as well as individuals. We can therefore compare the measure we have introduced to the standard measures of node and also of *group centrality*. In table 2 we report the obtained node centrality scores. All the measures assign the first score to monkey 3, second score to monkey 12, and third score to

Table 2. Node centralities: for each monkey of figure 2 we report age and sex group, information centrality C^{I} and the three standard centrality measures C^{D}, C^{C} and C^{B}. The flow betweenness centrality C^{F} is also reported in the last column.

	Age goup	Sex	C^{I}	C^{D}	C^{C}	C^{B}	C^{F}
1	14–16	M	0.139	0.211	0.134	0.006	0.084
2	10–13	M	0.000	0.000	0.050	0.000	0.000
3	10–13	M	0.375	0.684	0.143	0.260	0.444
4	7–9	M	0.131	0.158	0.133	0.000	0.023
5	7–9	M	0.123	0.105	0.132	0.000	0.044
6	14–16	F	0.000	0.000	0.050	0.000	0.000
7	4–5	F	0.131	0.158	0.133	0.000	0.027
8	10–13	F	0.131	0.158	0.133	0.003	0.073
9	7–9	F	0.115	0.053	0.131	0.000	0.000
10	7–9	F	0.131	0.158	0.133	0.000	0.018
11	14–16	F	0.123	0.105	0.132	0.000	0.004
12	10–13	F	0.180	0.474	0.139	0.060	0.219
13	14–16	F	0.156	0.316	0.136	0.011	0.115
14	4–5	F	0.139	0.211	0.134	0.000	0.028
15	7–9	F	0.156	0.316	0.136	0.011	0.115
16	10–13	F	0.000	0.000	0.050	0.000	0.000
17	7–9	F	0.131	0.158	0.133	0.000	0.018
18	4–5	F	0.000	0.000	0.050	0.000	0.000
19	14–16	F	0.000	0.000	0.050	0.000	0.000
20	4–5	F	0.000	0.000	0.050	0.000	0.000

monkeys 13 and 15. The six isolated monkeys are the least central nodes according to C^{I}, C^{D} and C^{C}. C^{B} assigns a zero score to fourteen nodes, the six isolated monkeys, and nodes 4, 5, 7, 9, 10, 11, 14, 17. In fact, the latter nodes do not contribute to the shortest paths, although they have a degree equal to or larger than one (for instance node 14 has four neighbours). Conversely, the flow betweenness C^{F}, assigns a zero score to seven nodes: the six isolated nodes and node 9. For the data set considered, the ranking of the 20 nodes produced by C^{I} is the same as that produced by C^{D} and C^{C}. Nevertheless, as shown in table 2, the values obtained are different. For instance the first node in the rank, namely monkey 3, has $C^{\mathrm{D}} = 0.684$, $C^{\mathrm{I}} = 0.375$ and $C^{\mathrm{C}} = 0.143$. In figure 3 we plot the centrality score of each node, normalized to the highest score. The nodes are ordered as a function of their normalized score based on C^{I}. The general behaviour of C^{I} is similar to that of C^{D}, C^{C} and C^{B}, although for the first four nodes the normalized values of C^{I} are smaller than C^{D} and C^{C}, and larger than C^{B} and C^{F}. The flow betweenness shows some differences with respect to the other measures: the two peaks at rank 9 and rank 12, corresponding respectively to node 8 and node 5, indicate that these two monkeys rank higher, according to C^{F}.

We now consider six different groups, four formed by age and two formed by sex [19]. Group 1 contains the five monkeys having age 14–16, group 2 the five monkeys having age 10–13, group 3 the six monkeys having age 7–9 and group 4 the four monkeys having age 4–5. Group 5 is made by the five females, while group 6 is made by the fifteen males. As illustrated in

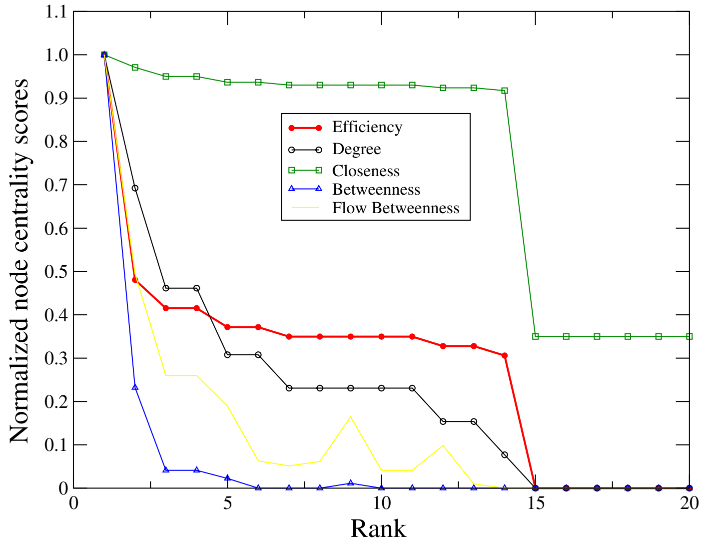

Figure 3. Node centrality score for the 20 nodes of the graph of interactions amongst monkeys plotted in figure 2. The values reported are normalized to the highest score. The nodes are ordered according to their value of C^{I} (see table 2). Node normalized to the highest score.

figure 4, among the age groups the most central one is the 10–13 years old (group 2), according to all the four measures. This is the group containing monkey 3, who is also the most central node as an individual. The four age groups in decreasing order of importance are: 2, 3, 1, 4 for the information centrality, 2, 1, 3, 4 for the degree centrality, 2, 1, 4, 3 for the closeness centrality and 2, 1, 3–4 for the betweenness centrality which assigns a score equal to zero to the two youngest groups. C^{I} is the only measure assigning the second position to the 7–9 years old, while the other three measures assign the second position to the 14–16 years old. C^{I} assigns last position to group 4 (age 4–5), similarly to the degree centrality and to the betwenness centrality. Among the sex groups the most central one is the male group (which is also the largest one) for both C^{I} and C^{D}. The situation is inverted according to C^{B}, while C^{C} attributes the same score to the two groups.

3.3. Large networks

In this section we investigate how the information centrality is statistically distributed among the nodes of large graphs. In order to reduce the statistical fluctuations, we have computed the cumulative distribution $P(C^{\mathrm{I}})$ defined in terms of the (differential) distribution $p(C^{\mathrm{I}})$ as:

$$P(C^{\mathrm{I}}) = \int_{C^{\mathrm{I}}}^{+\infty} p(X)\mathrm{d}X = \int_{C^{\mathrm{I}}}^{+\infty} \frac{N(X)}{N}\,\mathrm{d}X, \qquad (8)$$

where $p(X)\mathrm{d}X$ is the probability to find a node with an information centrality ranging in the interval X, $X + \mathrm{d}X$, while $N(X)\mathrm{d}X$ is the number of nodes with an information centrality ranging in the interval X, $X + \mathrm{d}X$, and N is the total number of nodes in the graph.

First, we have considered two kinds of artificial generated graphs, namely: *Erdős–Rényi (ER) random graphs*, and *generalized random graphs with a given degree distribution*. We

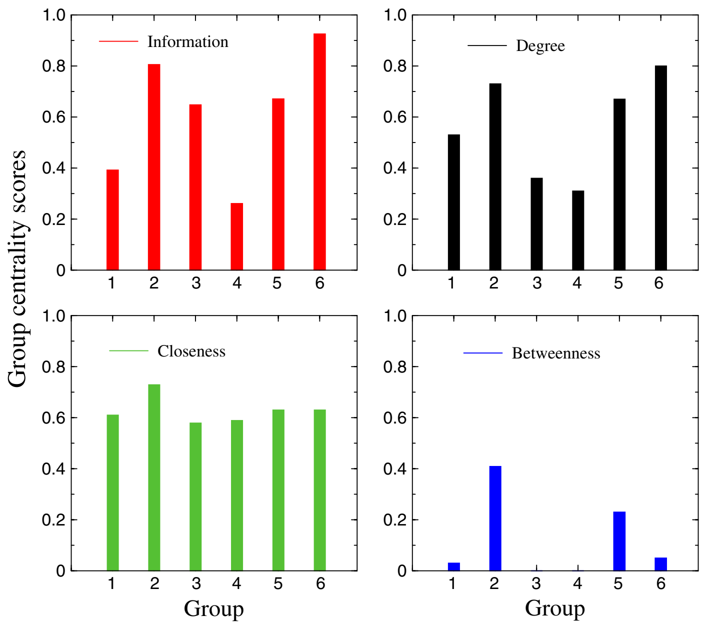

Figure 4. Group centrality scores for the four centrality measures and for each of the six groups considered, namely: group 1 (age 14–16), group 2 (age 10–13), group 3 (age 7–9), group 4 (age 4–5), group 5 (males), group 6 (females).

have generated ER random graphs, $G_{\mathrm{ER}}(N, K)$, with $N = 1000$ nodes and $K = 5000$ links, and generalized random graphs with $N = 1000$ nodes and a power-law degree distribution $p_k \sim k^{-\gamma}$ with exponent $\gamma = 3$ [3]–[5]. In figure 5, panel (a) and (b) we report as circles the cumulative distributions of information obtained in the two cases. The dashed line in panel (a) is a Gaussian fit to the points, while the dashed straight line in the log–log plot of panel (b) is a power law fit, $P(C^{\mathrm{I}}) \sim (C^{\mathrm{I}})^{-\mu}$, with an exponent $\mu = 1.75$. The latter result indicates that in a random graph with a power-law degree distribution, the information centrality is also distributed as a power law. In the case considered we have found: $p(C^{\mathrm{I}}) \sim (C^{\mathrm{I}})^{-2.75}$.

We have also considered two networks from the real world. The first graph describes the urban street pattern of the city of Richmond (US) [23]. The graph has $N = 697$ nodes representing street intersections and $K = 1086$ links representing streets. The obtained information distribution is reported in figure 5 panel (c). Also in this case, as in the case of the ER random graph shown in panel (a), C^{I} exhibits a single scale distribution. The dashed line in figure 5(c) is an exponential fit to the empirical distributions of the form $P(C) \sim \exp(-C/s)$ with $s = 0.0017$. Finally, we have considered in figure 5(d) the protein–protein interaction network of *S. cerevisiae*. The graph consists of $N = 1870$ nodes, the proteins, connected by $K = 2240$ links representing physical interactions among proteins [24]. This network exhibits a broad-scale information distribution as the artifical network considered in panel (b). The dashed line in the log–log plot of figure 5(d) is a power law fit to the empirical distribution, $P(C) \sim C^{-\mu}$, with $\mu = -1.52$.

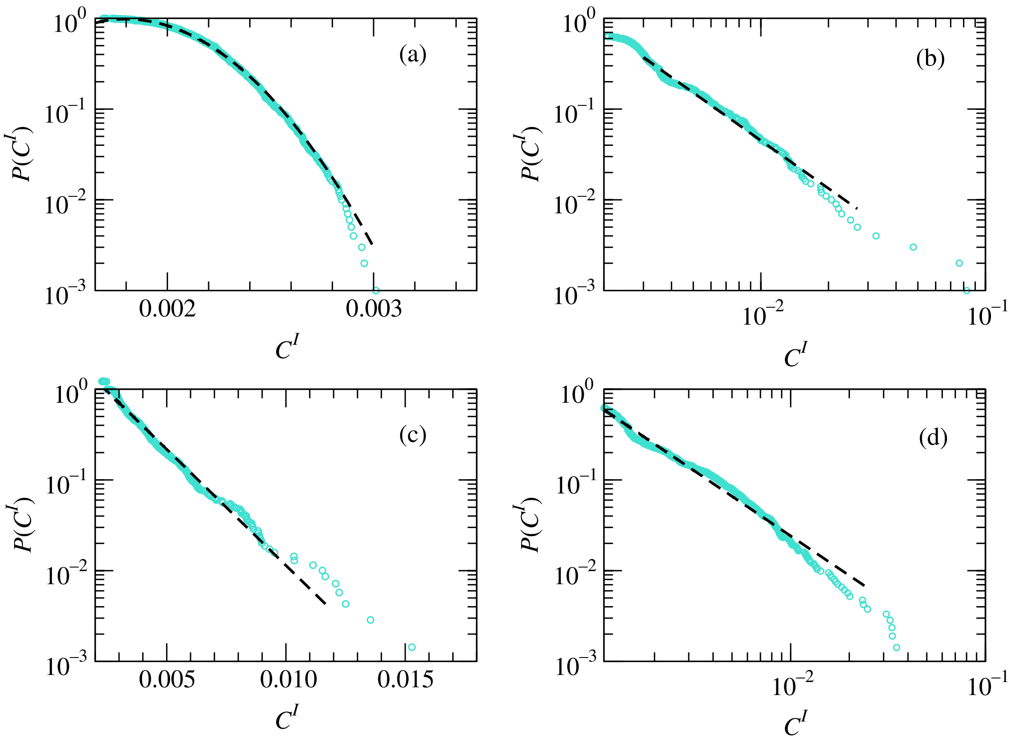

Figure 5. Cumulative distributions of information centrality in four large networks, namely: (a) ER random graph with $N = 1000$ nodes and $K = 5000$ links; (b) generalized random graph with $N = 1000$ nodes and a scale-free degree distribution $p_k \sim k^{-3}$; (c) the urban street network of the city of Richmond (US) with $N = 697$ and $K = 1086$; (d) the *S. cerevisiae* protein–protein interaction network with $N = 1870$ nodes and $K = 2240$ links. The results in panels (a) and (b) are averages over an ensemble of 30 graphs. The dashed lines in panels (a) and (c) are respectively Gaussian and exponential fits to the distributions, while the dashed lines in panels (b) and (d) are power law fits.

4. Conclusions

In this paper, we have introduced a new class of centrality measures, the so-called delta centrality, that extends in a natural way to groups. In particular, we have focused on a measure of this class, the information centrality C^{I}, that is based on the network efficiency. We have illustrated similarities and dissimilarities with respect to the standard measures adopted in sociometry by considering some small networks. We have also investigated how the information centrality is statistically distributed among the nodes of large graphs, by considering artificial generated graphs and networks from the real world. C^{I} is the first step toward the idea of measuring network topology through network function and dynamics [25, 26]. C^{I} has already proved useful in algorithms to find community structures [20], in the characterization of planar graphs [27]–[29], and to quantify the relevance of soluble mediators in the human immune cell network [30]. It remains to be seen, in the light of further empirical work, if and in which cases the new measure is more appropriate than the others.

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