Complex Networks (MTH6142)

2023/24

WEEK 1 Lecture 1

- **Module Convenor**: Prof Vito Latora

- **Lectures:**
  - Wednesday 11:00-13:00 (2 hours)
  - Friday 11:00-12:00 (1 hour)

- **Tutorials (from week 2):**
  - Friday 12:00-13:00 (1 hour)

- **Online forum:** Please first ask questions here

- **Office hours:**
  - Monday 15:00-16:00,
  - Wednesday 13:00-14:00 (Learning Café, Room MB-B11)

Module Assessment

- Assessed courseworks: worth 20%
  - 5 courseworks (3 Quizzes, 2 Handwritten)

  each worth 4% of the final mark (they open on a Friday and close at 5 pm on the following Wednesday: deadline in weeks 3, 5, 7, 9, 11).

- Final exam: worth 80% of the final mark.

  Handwritten exam.

  Further details available later.

10 Formative assignments:

Every week (except week 1) to be discussed in the **tutorial** hour

Typed lecture notes (written by Prof Ginestra Bianconi) available from QMPlus.

- **Handwritten notes and slides:** Available on QMPlus after the lecture/tutorial

- **Useful textbooks**

  -Newman, Networks: An Introduction, Oxford University Press 2010

  -Barabasi Network Science Cambridge University Press 2016

  -Latora, Nicosia, Russo, Complex Networks Cambridge University Press 2017

Complex Networks

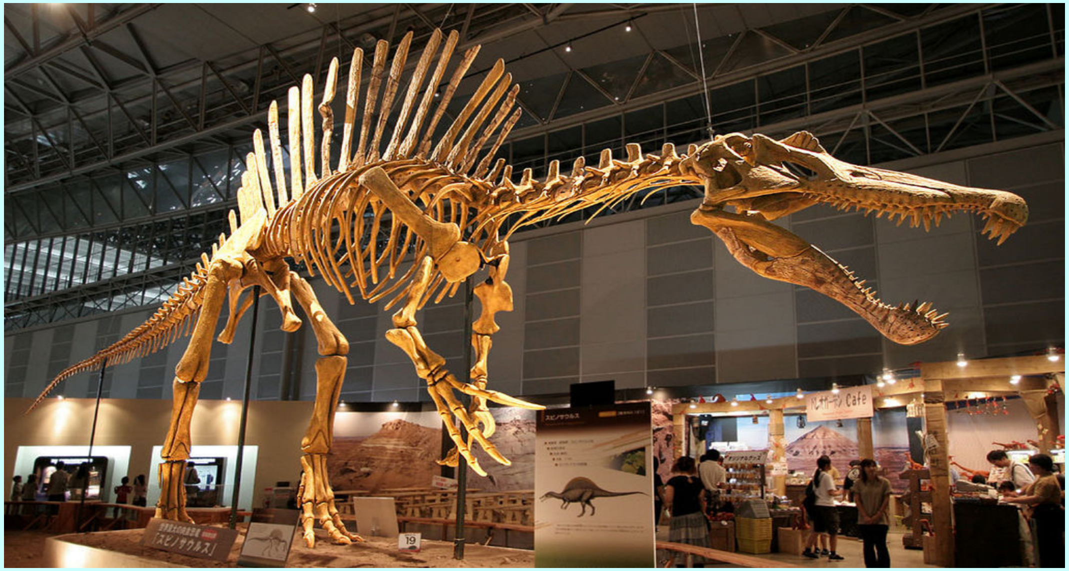
Introduction

Vito Latora

School of Mathematical Science, Queen Mary Univ of London
and
Dipartimento di Fisica, Universita' di Catania

Complex networks…

..i.e. what can we learn of a complex system by looking at the backbone of its interactions ?



Two examples of Complex Systems

Example 1: Bird flocks

Definitions of Complex System

We have many individuals plus their interactions…..
…..or even more than that !!

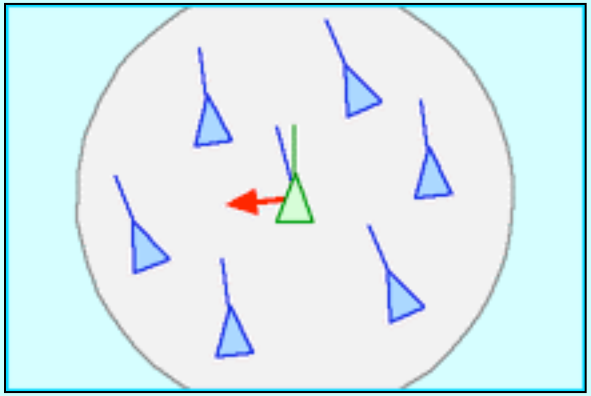
"More is different"
Anderson, Science 1972

In a complex system, simple rules give rise to complex behaviors

In a complex system, simple rules give rise to complex behaviors

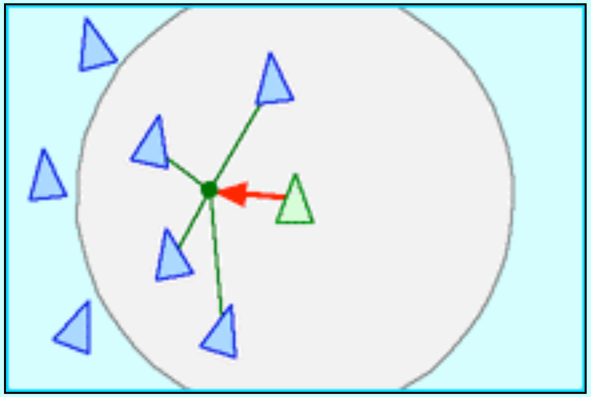
Reynolds (1986): Flocking model

ALIGNMENT



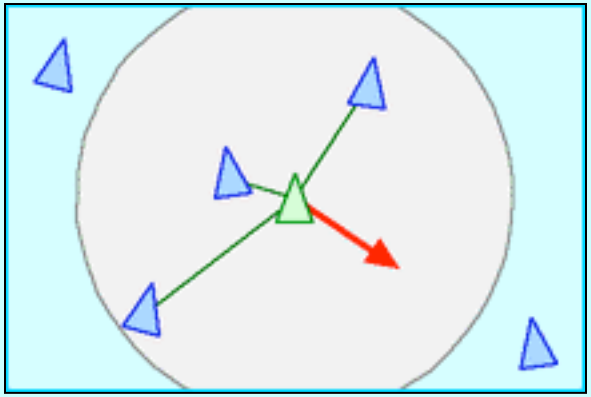
Steer to the average heading of local flockmates

COHESION



to move towards the average position of local flockmates

SEPARATION



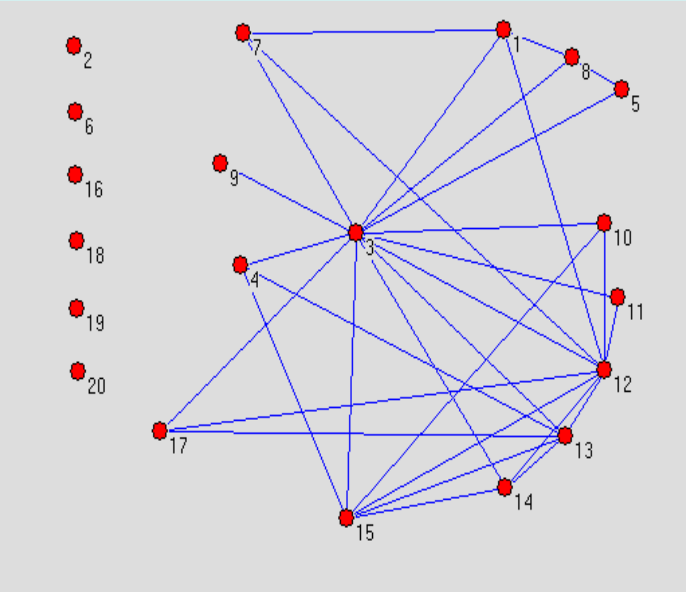
Steer to avoid crowding local flockmates

Vicsek et al. PRL 75(1995) 1226

Ballerini et al. PNAS 105(2008)1232

INTERACTIONS are the MAGIC INGREDIENT !!

We represent the interactions in a complex system as a complex network !!!



Three months of primate interactions
L. Wolfe (1992)

1) The animals are the nodes of the network

2) The interactions are the links of the network (can be weighted, directed, time-varying, etc..)

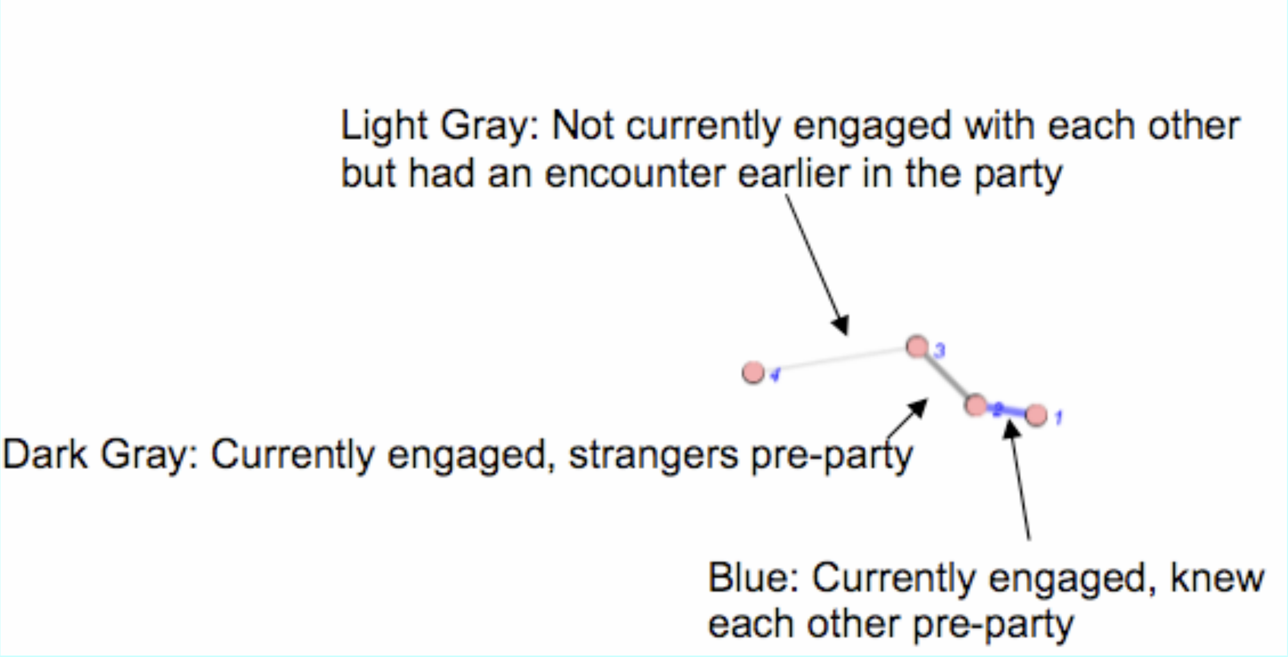
3) Networks are usually sparse, and neither regular nor random

Example 2:


slice:1 time:1.500-31.500
layout:Multiple component Kamada-Kawai layout optimum distance: 30.0minimum epsilon: 1.0cool factor: 0.25initial KK energy: 4265.800017385696
33
11

Example 2: The life of the party (Moody et al. 2005)

- A social network changing over time
- Kamada-Kawai (springs)
- Two types of nodes, three types of links



The life of the party (Moody et al. 2005)

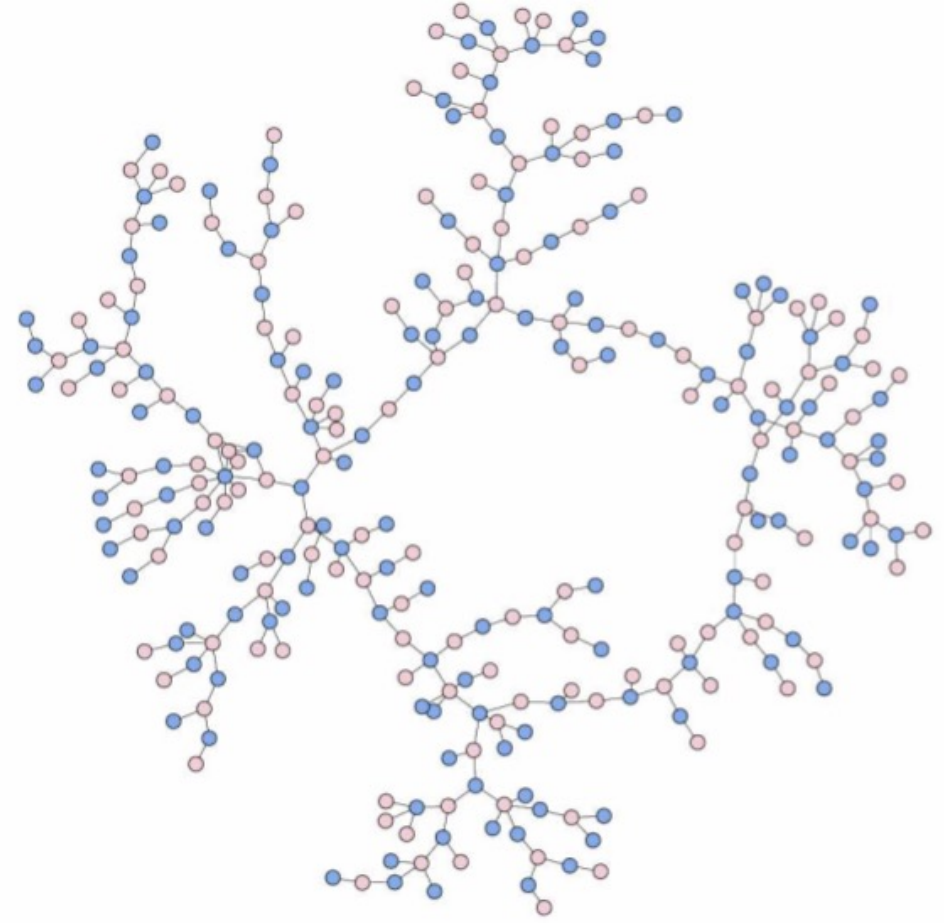
"So far I'm doing all right. I went over to talk to six people, but nine people have come over to talk to me."

- Encounters shrink social distances
- <u>95% of encounters with pre-known</u>
- Bridging (n=24)
- Outside the party network (n=18)
- n=40 knows only 10 (av=31). Ends up with 23 new
- Homophilous pairings (n=30 + 31)
- High physical attractiveness (n63) vs low (n82)
- High self-monitors (size) (n=79)

It is an exciting topic because…

..networks are everywhere !!

Social Networks

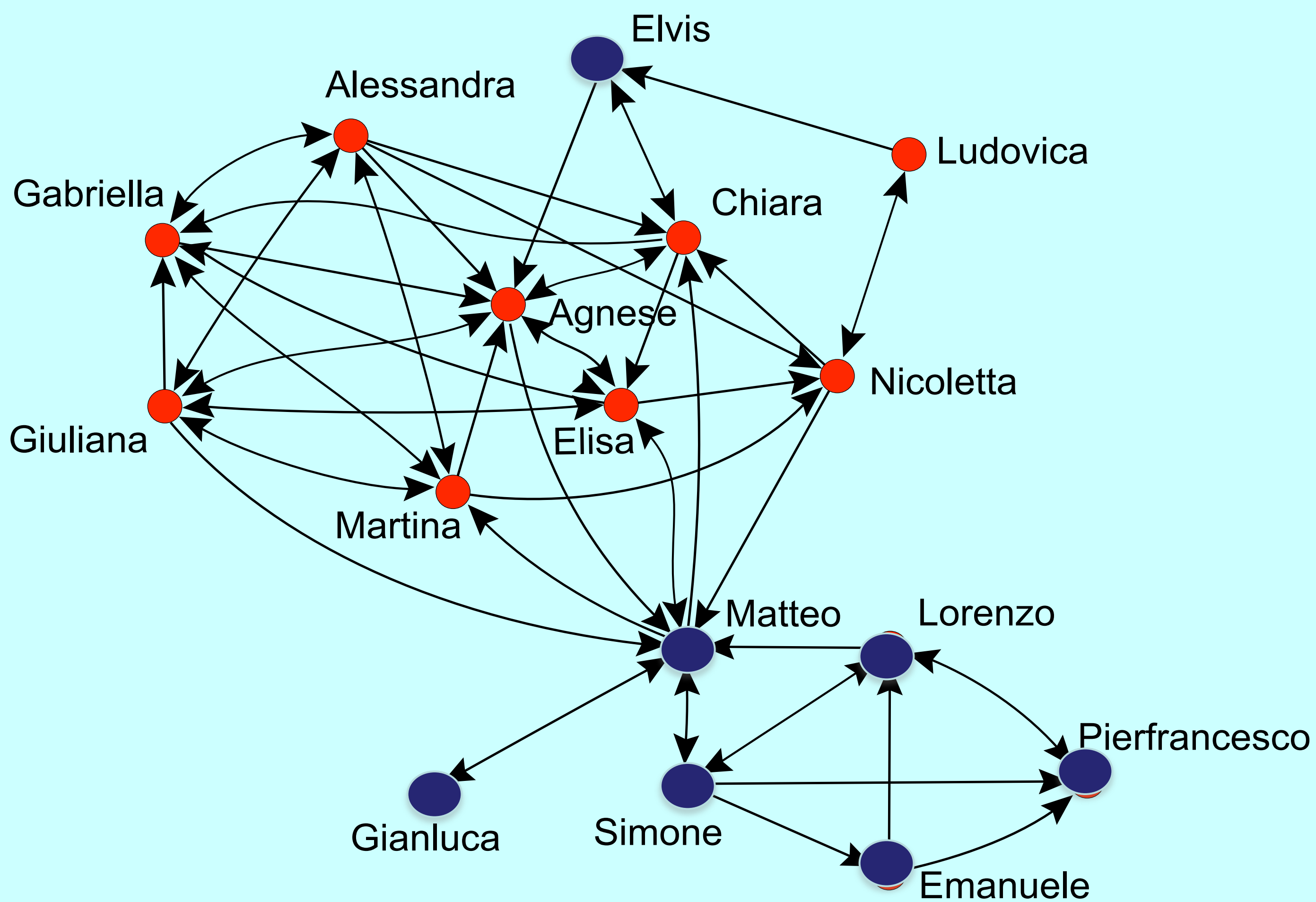


P S Bearman, J Moody, and K Stovel, Am J Soc (2004).

Relationships in high school

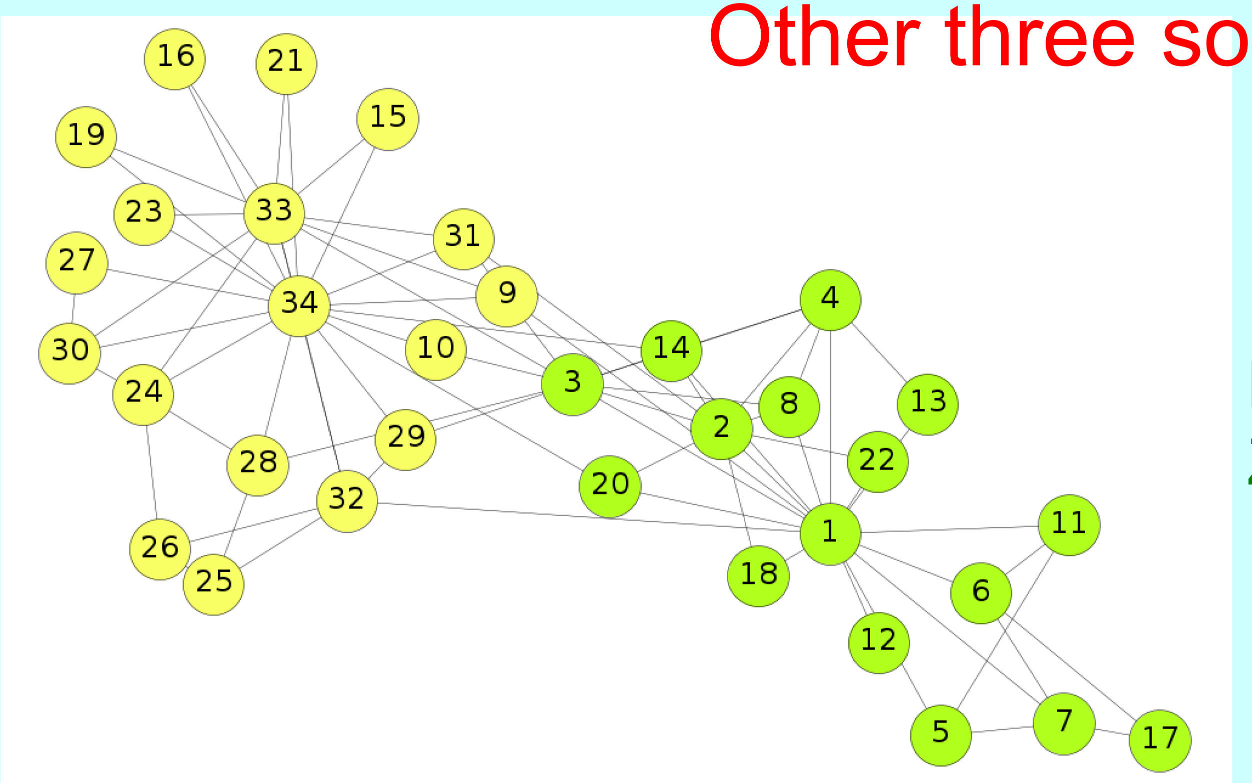
...You can construct yourself your home-made social network !!!

Social Networks



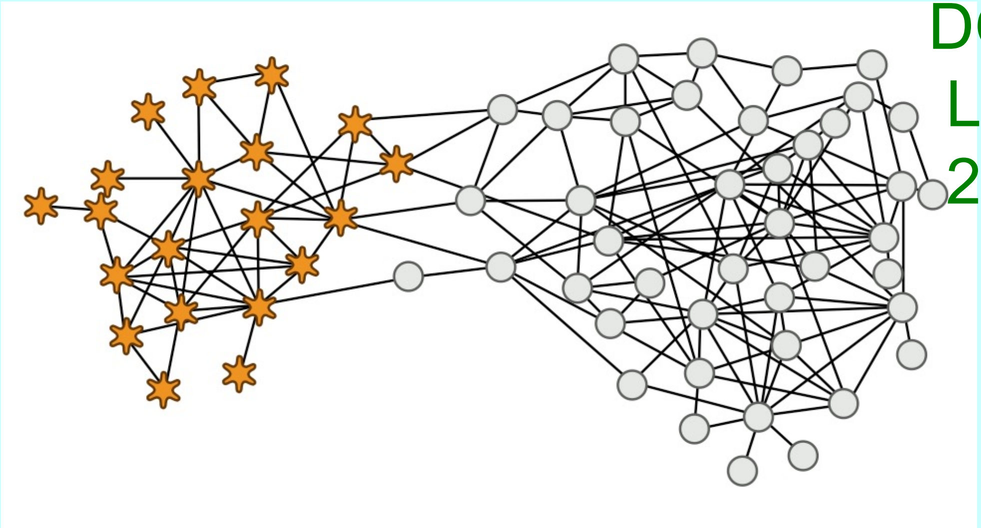
Friendships at the kindergarten of my daughter Elisa (2006)

Other three social examples

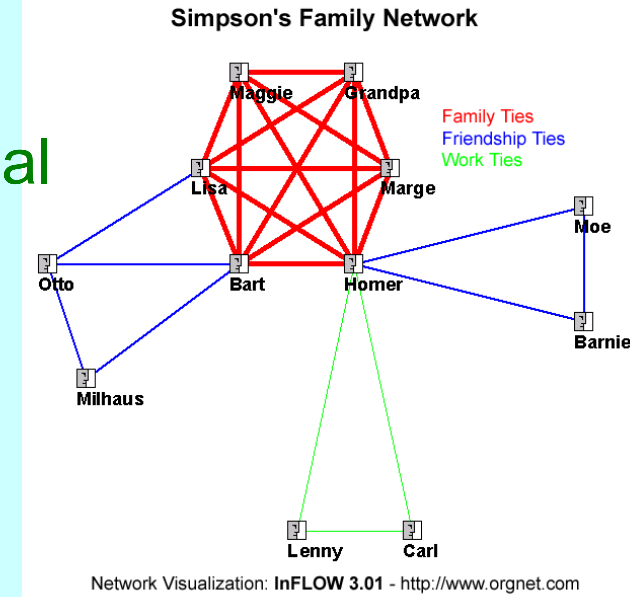


KARATE CLUB
Zachary 1977

DOLPHINS
Lusseau et al
2003

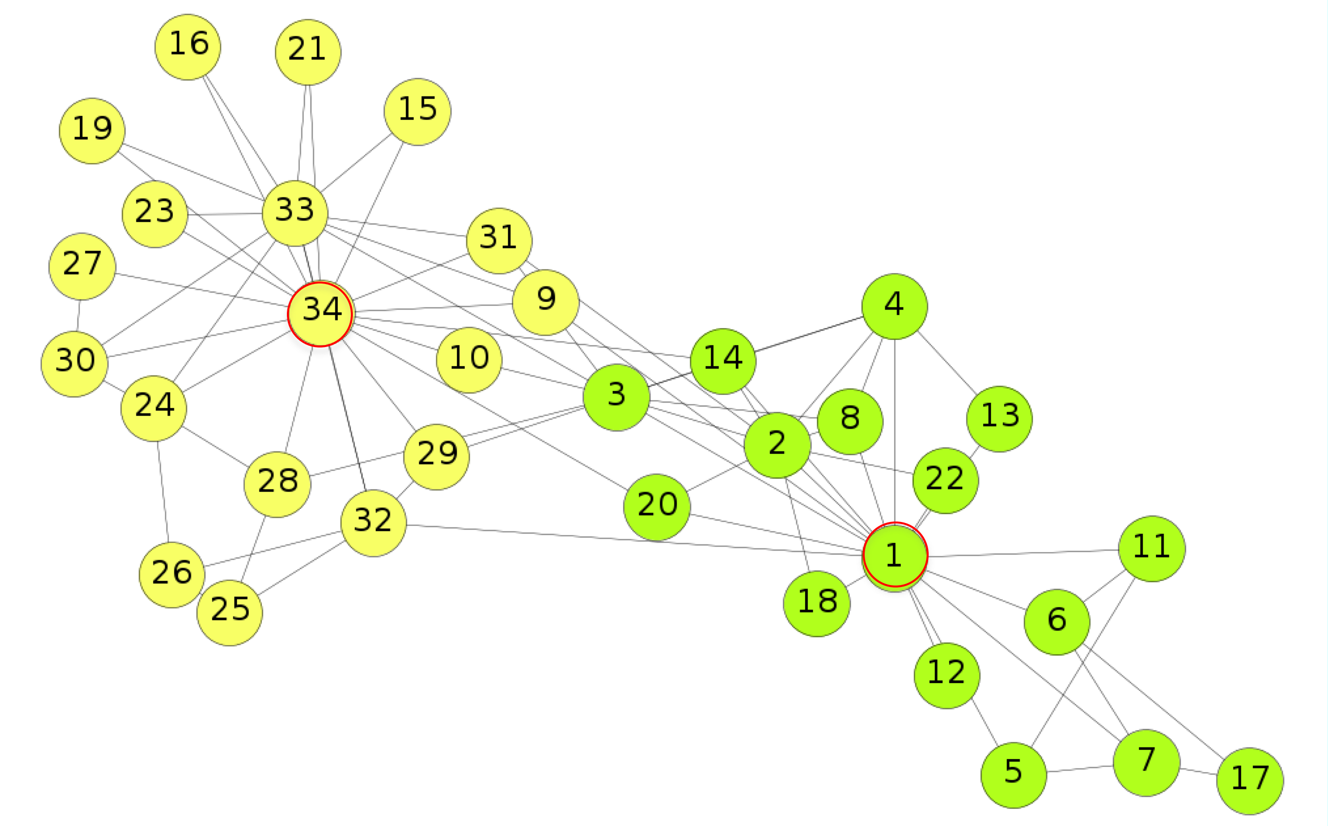


COMICS



Zachary's karate club

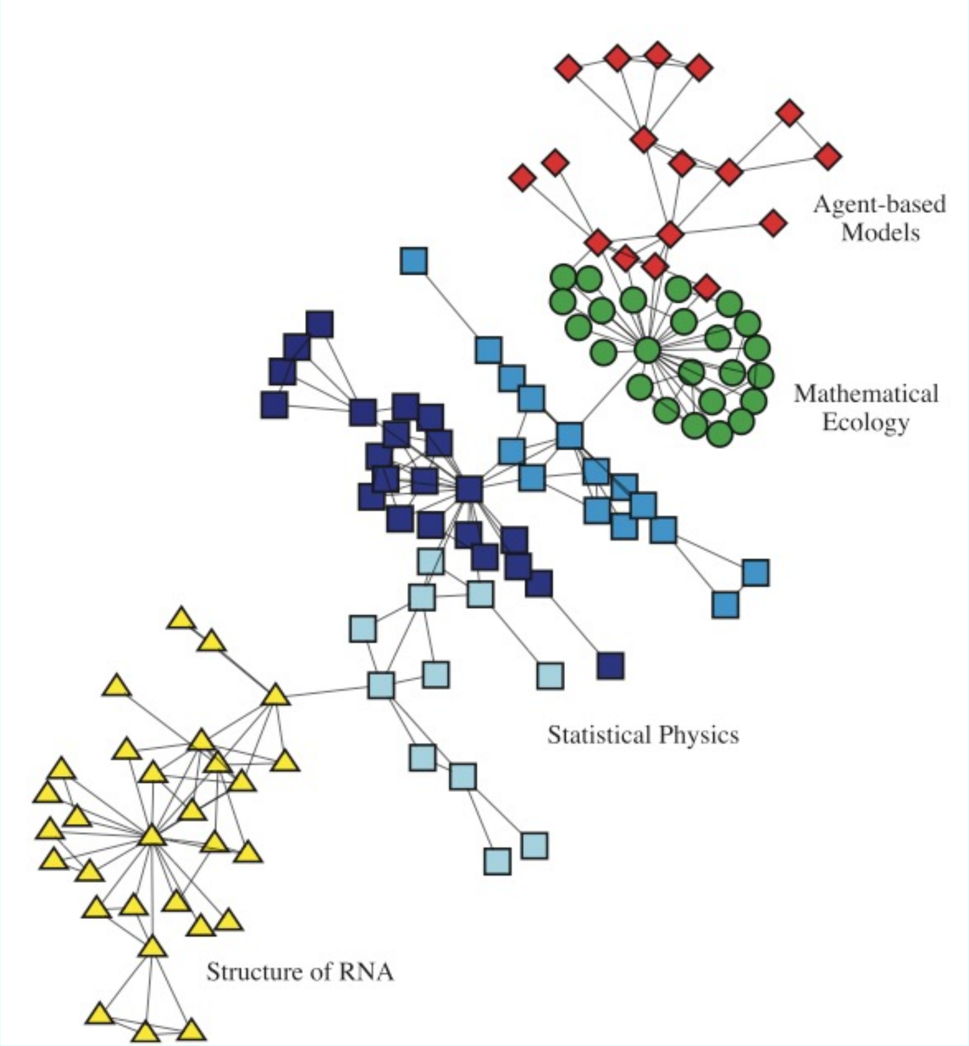
Outside club activities (3 years of observations before fission by anthropologist Wayne Zachary)



Fission:16 members following 1 (the instructor Mr. Hi),

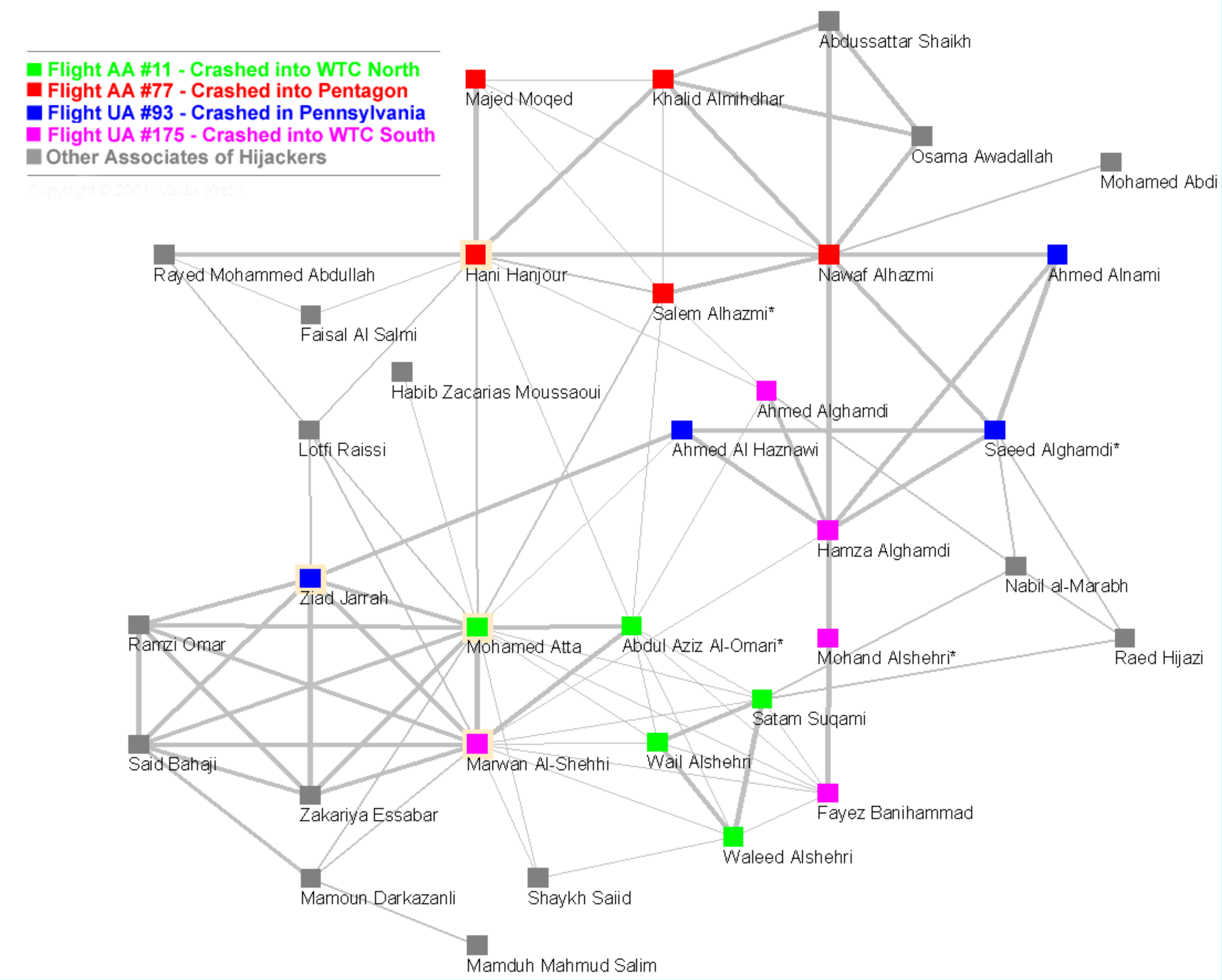
18 members following 34 (Mr. John A. the administrator)

The good and the bad collaboration net



Researchers at the Santa Fe Institute, Newman PNAS 04

The good and the bad collaboration net



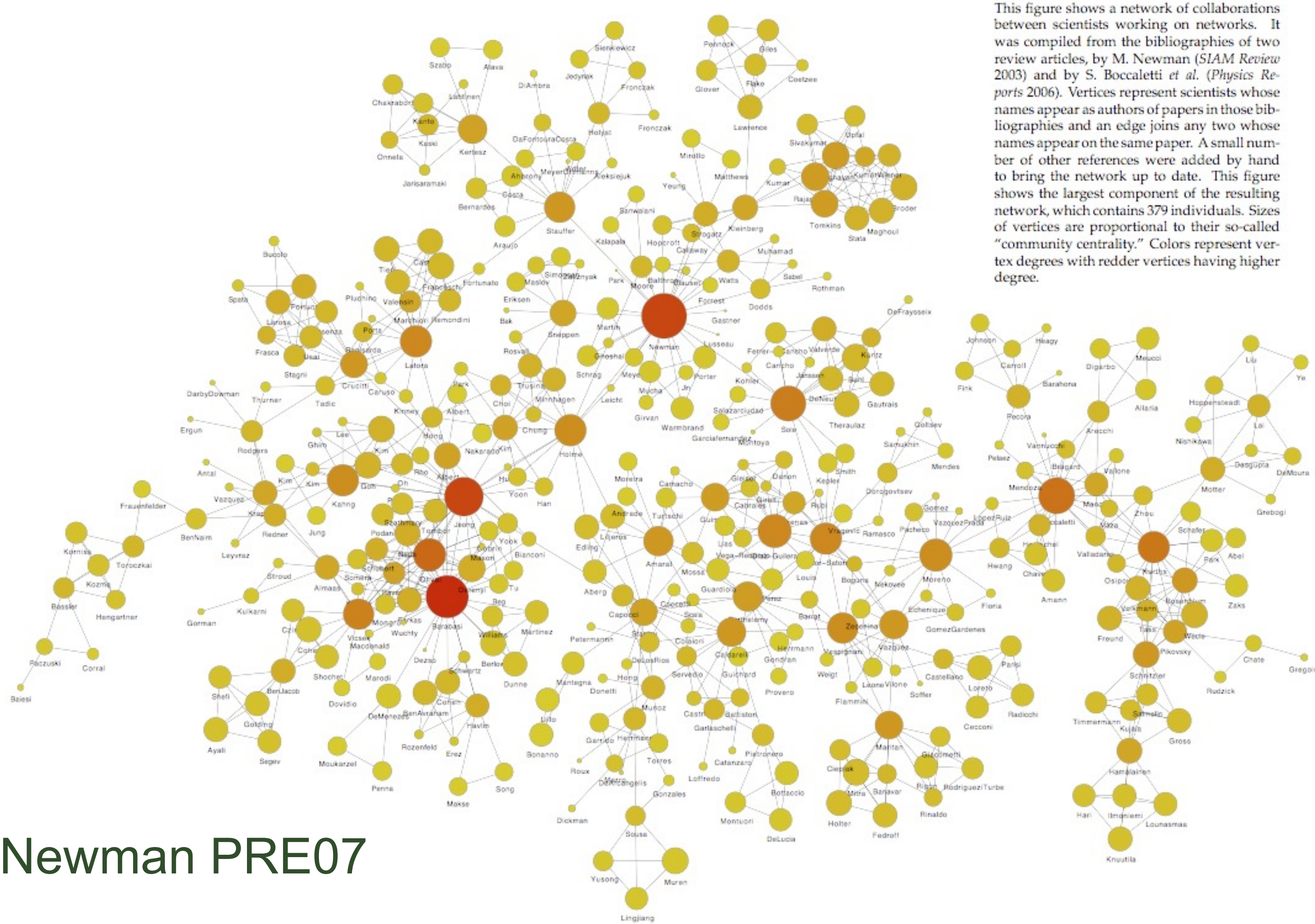
Weighted network

Krebs, Connections 2001

Collaboration network of scientists working on networks !!!

Collaborations Between Network Scientists

This figure shows a network of collaborations between scientists working on networks. It was compiled from the bibliographies of two review articles, by M. Newman (*SIAM Review* 2003) and by S. Boccaletti *et al.* (*Physics Reports* 2006). Vertices represent scientists whose names appear as authors of papers in those bibliographies and an edge joins any two whose names appear on the same paper. A small number of other references were added by hand to bring the network up to date. This figure shows the largest component of the resulting network, which contains 379 individuals. Sizes of vertices are proportional to their so-called "community centrality." Colors represent vertex degrees with redder vertices having higher degree.



Newman PRE07

Happiness is……

Fowler, Christakis. 2008

``Dynamic spread of happiness in a large social network: longitudinal analysis over 20 years in the Framingham Heart Study.´´

British Medical Journal 337, no. a2338: 1-9

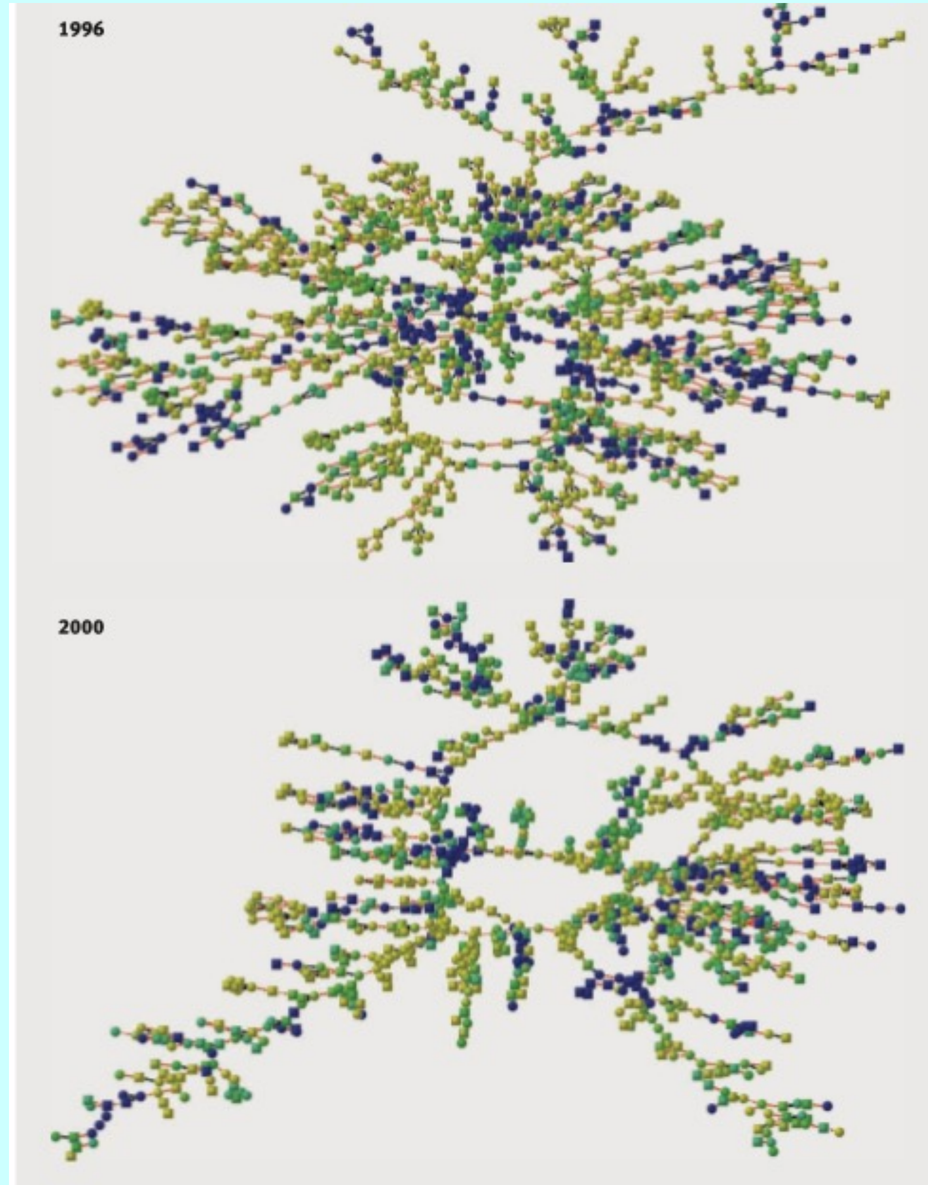


Fig 1 | Happiness clusters in the Framingham social network. Graphs show largest component of friends, spouses, and siblings at exam 6 (centred on year 1996, showing 1181 individuals) and exam 7 (year 2000, showing 1020 individuals). Each node represents one person (circles are female, squares are male). Lines between nodes indicate relationship (black for siblings, red for friends and spouses). Node colour denotes mean happiness of ego and all directly connected (distance 1) alters, with blue shades indicating least happy and yellow shades indicating most happy (shades of green are intermediate)

Happiness is……having happy friends !!

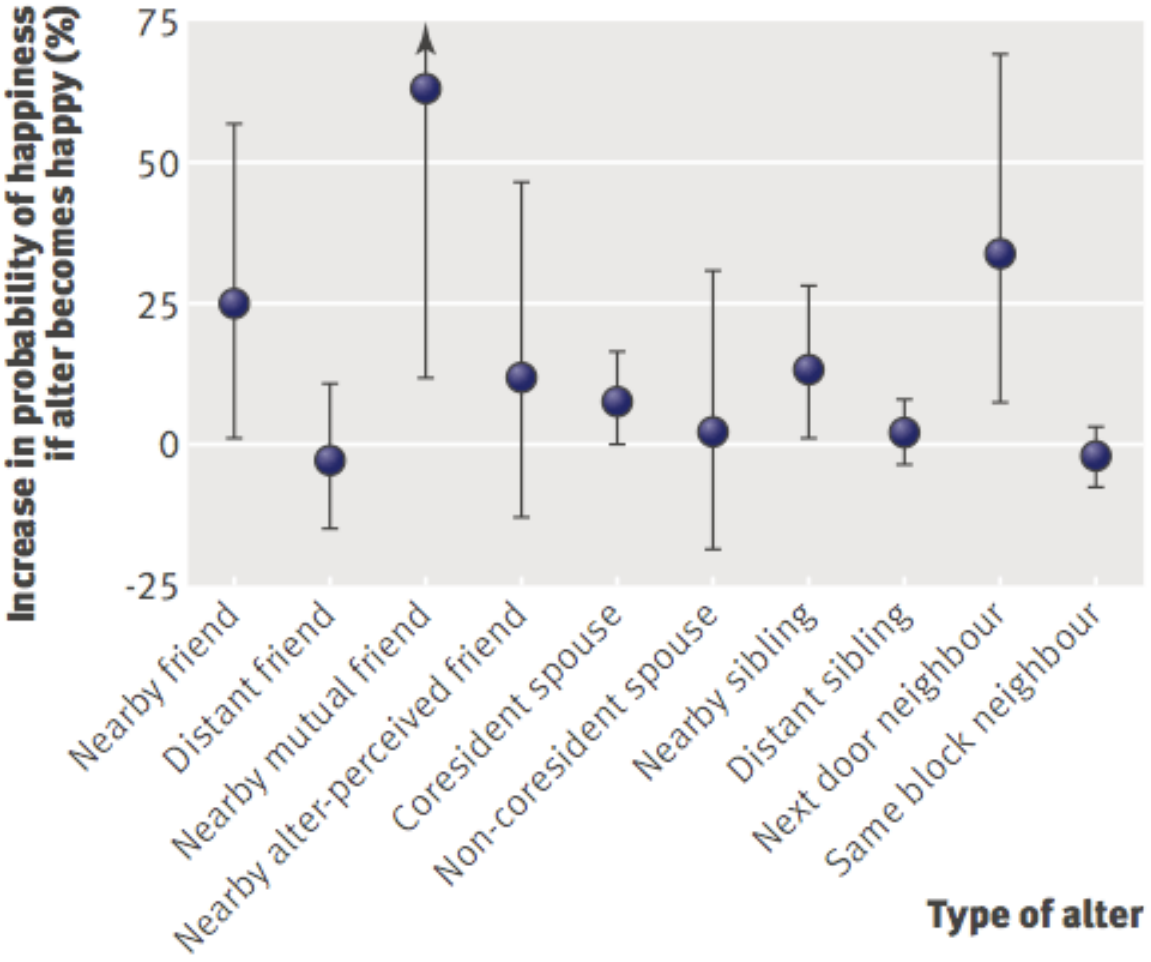
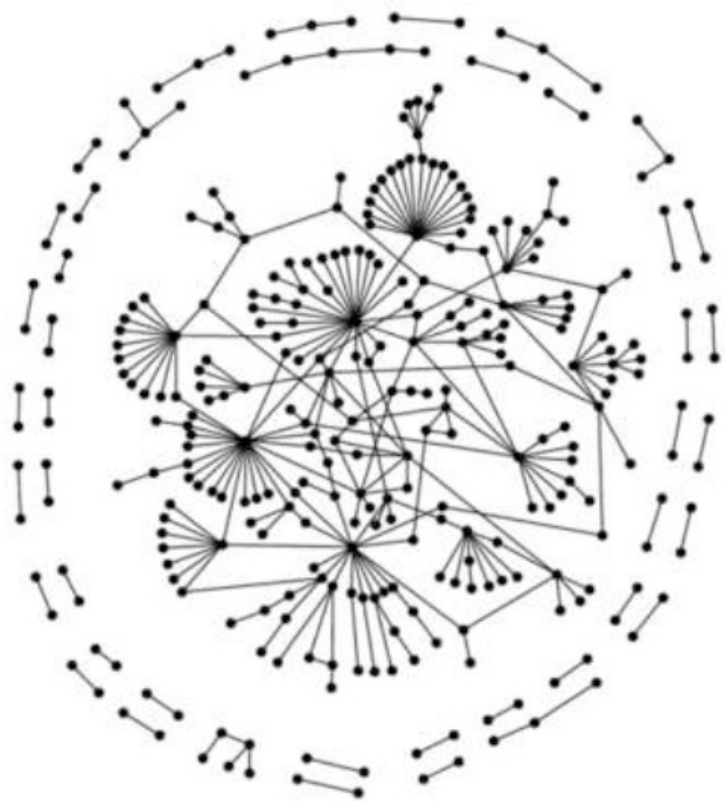


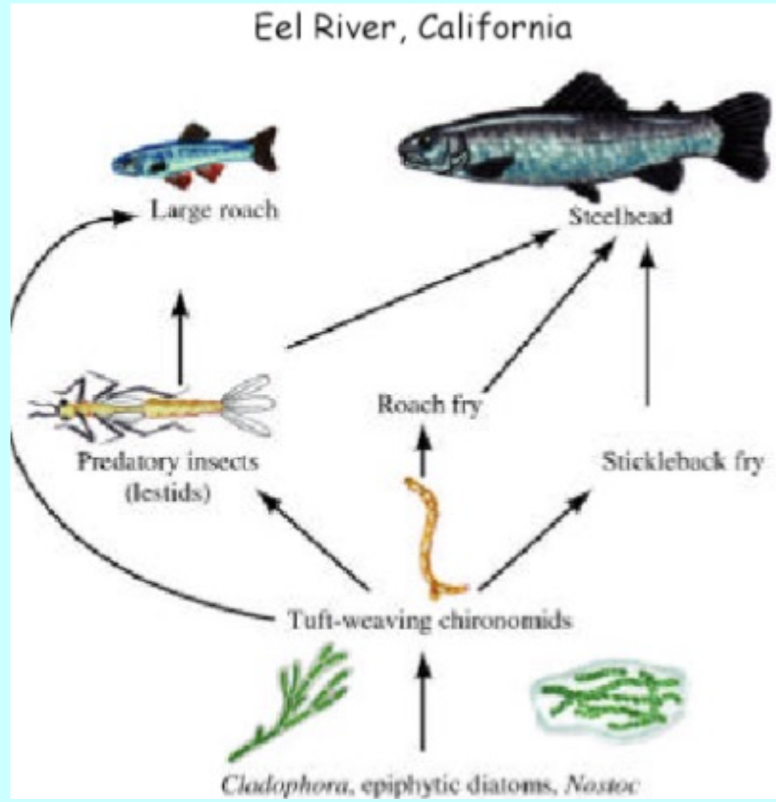
Fig 4 | Alter type and happiness in the Framingham social network. Friends, spouses, siblings, and neighbours significantly influence happiness, but only if they live close to

Tens of examples in biology



S Maslov and K Sneppen, Science (2002).

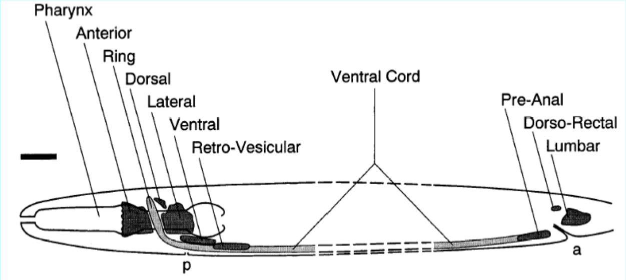
Protein interaction network in the Yeast (S.C.)

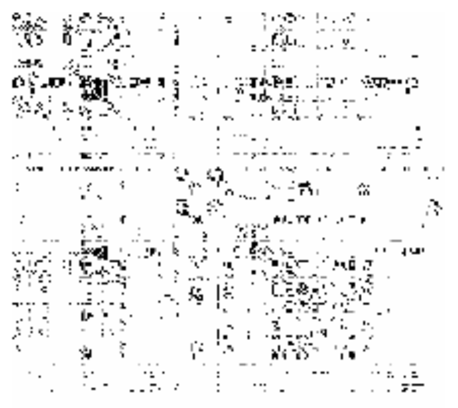


Food Webs

Brain networks: anatomical connectivity

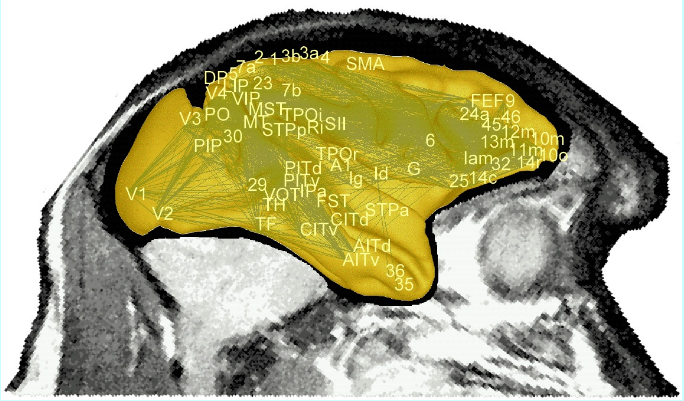
C. elegans: layout of ganglia

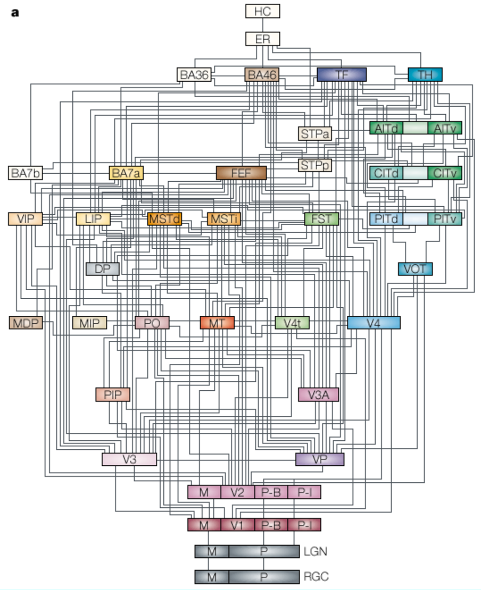




Brenner et al, 1975 mapped every single nervous cell, synapses and gap junctions

Neuron network



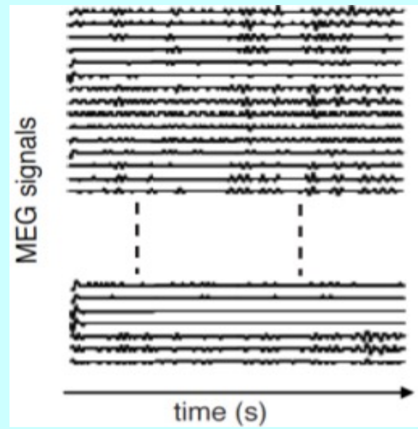


Cortical regions of macaque

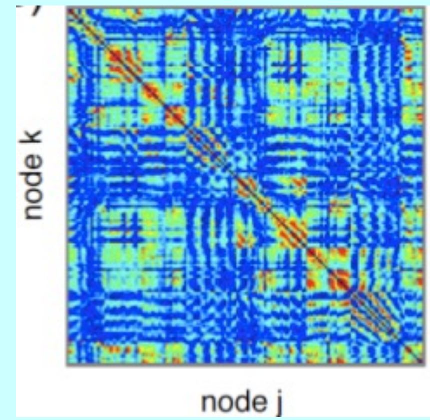
Links between cortical areas

Brain networks: functional connectivity

EEG, MEG, fMRI
signals



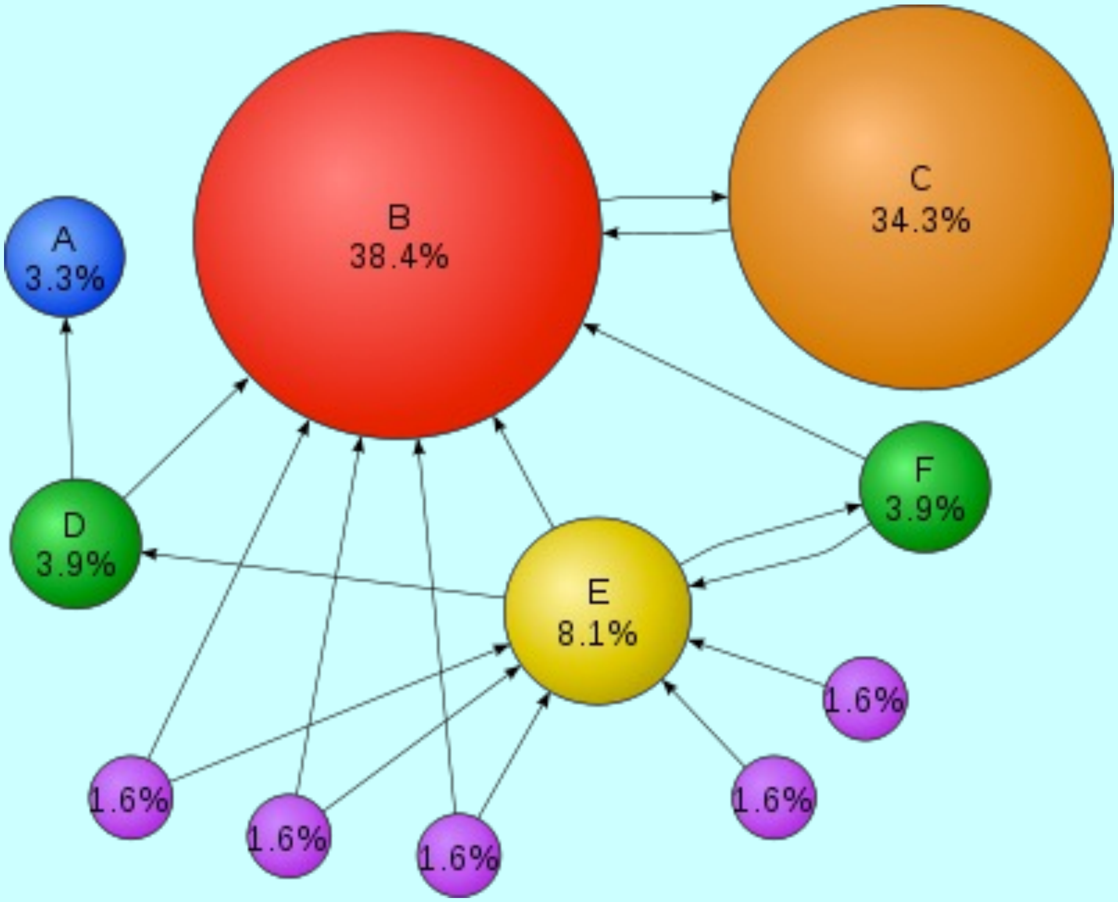
Coherence,
synchronization,
causality



Functional connectivity

Bullmore, Sporns, Nature Review Neuroscience 2009

Do not forget man-made systems !!



-Hyperlinks between Web pages

-- directed network

-- the largest net

>20 years of Complex Networks

Watts, Strogatz, Nature 393, 440 (1998)

Barabasi, Albert, Science 286, 509 (1999)

1) Characterize the structure of large real networks

2) Developing new models (growing graphs)

3) Dynamical processes (percolation, diffusion, spreading, games, network of dynamical systems)

Complex Networks (MTH6142)

- 1) Introduction
- 2) Basic properties
- 3) Centrality measures
- 4) Random graphs
- 5) Scale-free networks
- 6) Growing networks
- 7) Small-world networks
- 8) The configuration model

The small-world phenomenon

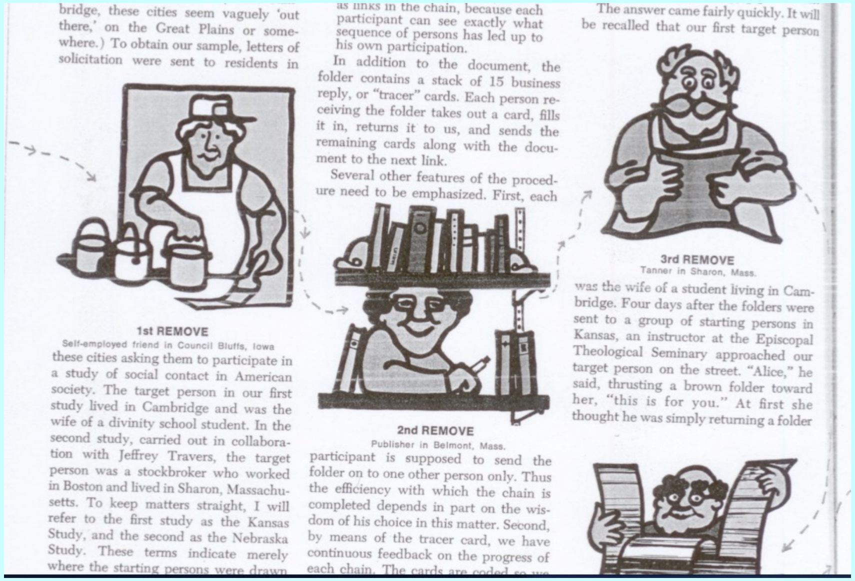
- FIRST experiment:

  S.Milgram, Psychology Today 2, 60 (1967)

  Of 170 chains started in Omaha, Nebraska, 126 dropped out and 44 reached destination

chain length	2	3	4	5	6	7	8	9	10
number of completed chains	2	4	9	8	11	5	1	2	2

Average distance 5.43



bridge, these cities seem vaguely 'out there,' on the Great Plains or somewhere.) To obtain our sample, letters of solicitation were sent to residents in

1st REMOVE
Self-employed friend in Council Bluffs, Iowa

these cities asking them to participate in a study of social contact in American society. The target person in our first study lived in Cambridge and was the wife of a divinity school student. In the second study, carried out in collaboration with Jeffrey Travers, the target person was a stockbroker who worked in Boston and lived in Sharon, Massachusetts. To keep matters straight, I will refer to the first study as the Kansas Study, and the second as the Nebraska Study. These terms indicate merely where the starting persons were drawn

as links in the chain, because each participant can see exactly what sequence of persons has led up to his own participation.

In addition to the document, the folder contains a stack of 15 business reply, or "tracer" cards. Each person receiving the folder takes out a card, fills it in, returns it to us, and sends the remaining cards along with the document to the next link.

Several other features of the procedure need to be emphasized. First, each

2nd REMOVE
Publisher in Belmont, Mass.

participant is supposed to send the folder on to one other person only. Thus the efficiency with which the chain is completed depends in part on the wisdom of his choice in this matter. Second, by means of the tracer card, we have continuous feedback on the progress of each chain. The cards are coded so we

The answer came fairly quickly. It will be recalled that our first target person

3rd REMOVE
Tanner in Sharon, Mass.

was the wife of a student living in Cambridge. Four days after the folders were sent to a group of starting persons in Kansas, an instructor at the Episcopal Theological Seminary approached our target person on the street. "Alice," he said, thrusting a brown folder toward her, "this is for you." At first she thought he was simply returning a folder

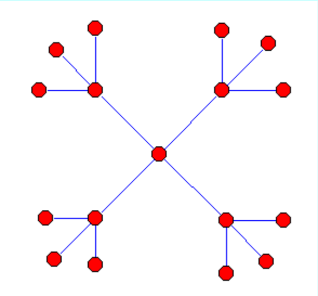
- Movie actors database N= 248243

0	1	2	3	4	5	6	7	8	∞
1	1181	71397	124975	25665	1787	196	22	2	23017

Average distance from Kevin Bacon 2.81

Average distance 3.65

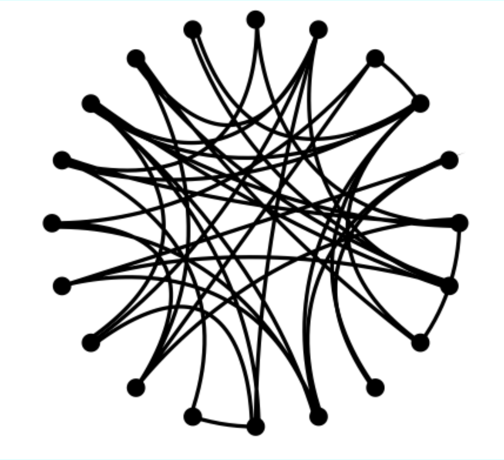
Small distances



k friends in 1 step, k^2 in 2 steps..

$$k^L \simeq N \qquad L \simeq \frac{\log N}{\log k} \sim \log N$$

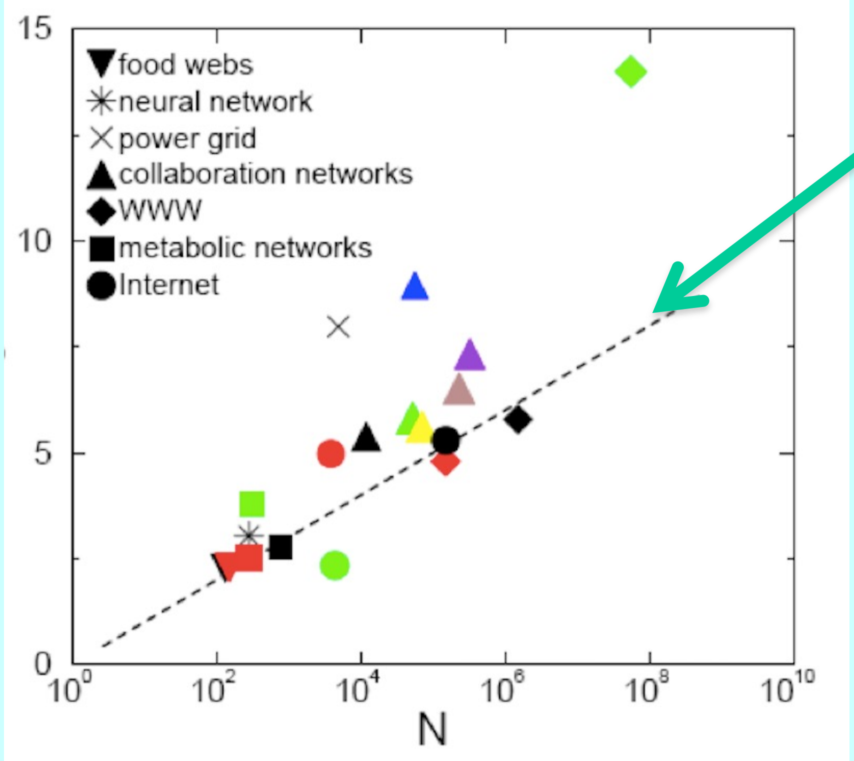
$$N = 10^9; k = 10^2 \rightarrow L \simeq 5$$



Random graphs

$$L_{rand} \approx \frac{\log N}{\log\langle k\rangle}$$

$$L \cdot \log\langle k\rangle$$



The problem is that your friends very often know each others !!

Clustering coefficient

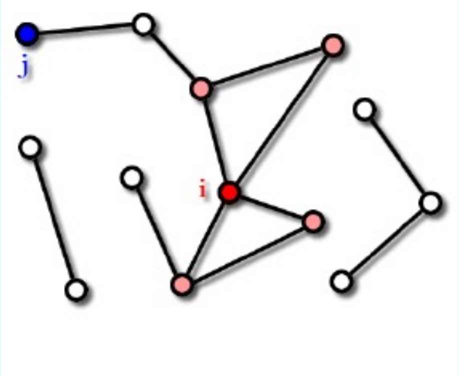
Social networks frequently contain cliques

$$C_{rand} \approx \frac{<k>}{N}$$

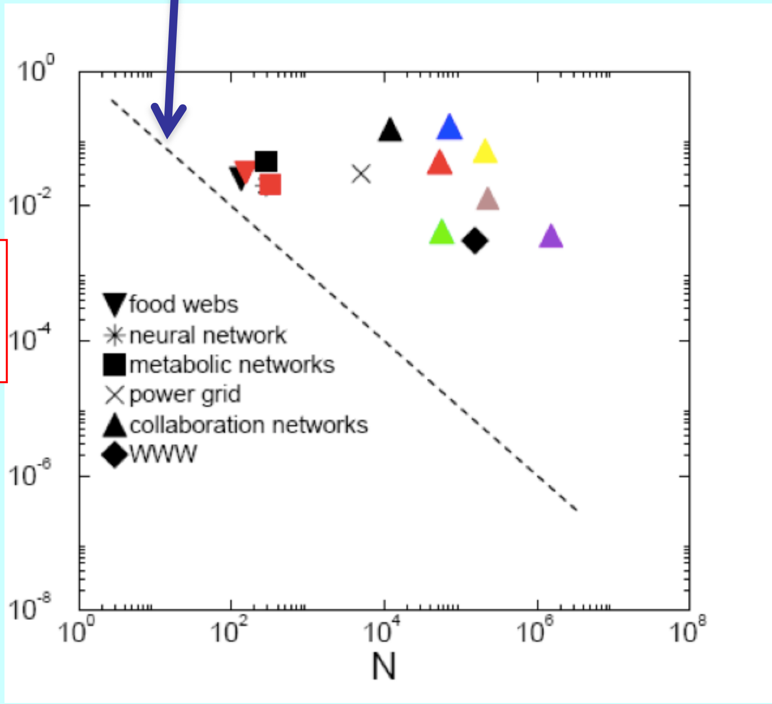
How close is the neighborhood of a node to a clique ?

$$C_i = \frac{\# \, links}{\frac{k_i(k_i - 1)}{2}}$$

$$C_i = \frac{2}{6} = \frac{1}{3}$$

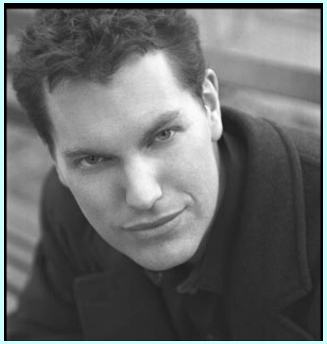


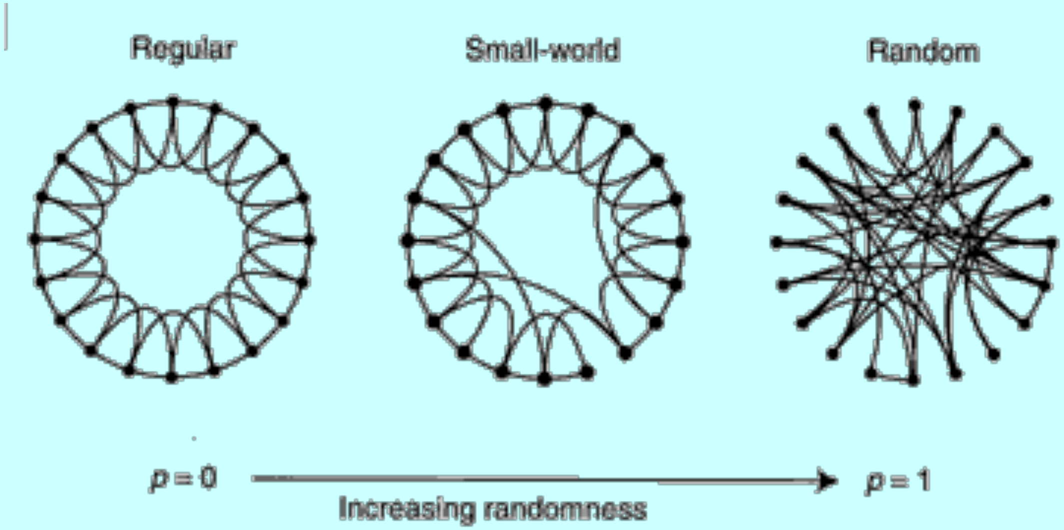
$$\frac{C}{<k>}$$

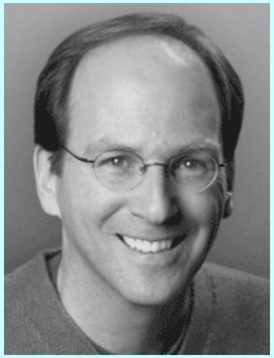


$$C = \frac{1}{N} \sum_i C_i \qquad 0 \leq C \leq 1$$

Watts-Strogatz (WS) small-world model

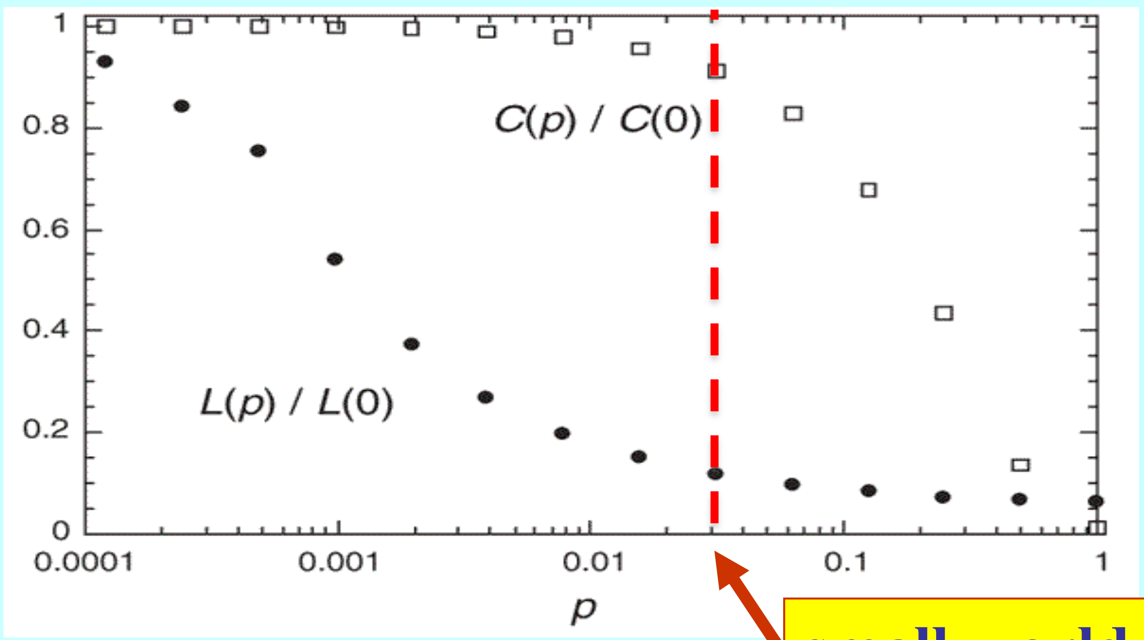






L ~ N
C = 1/2

L ~ log N
C ~ 1/N

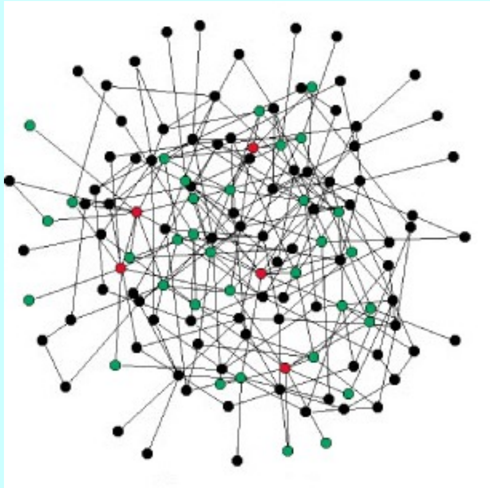


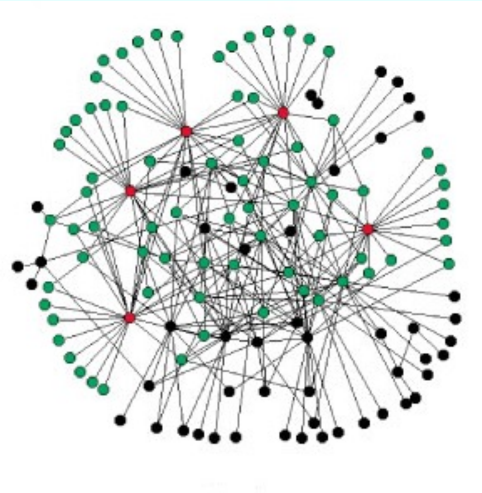
small-world behavior

Watts and Strogatz, Nature 339 (1998) 440

Degree distributions

Random graphs

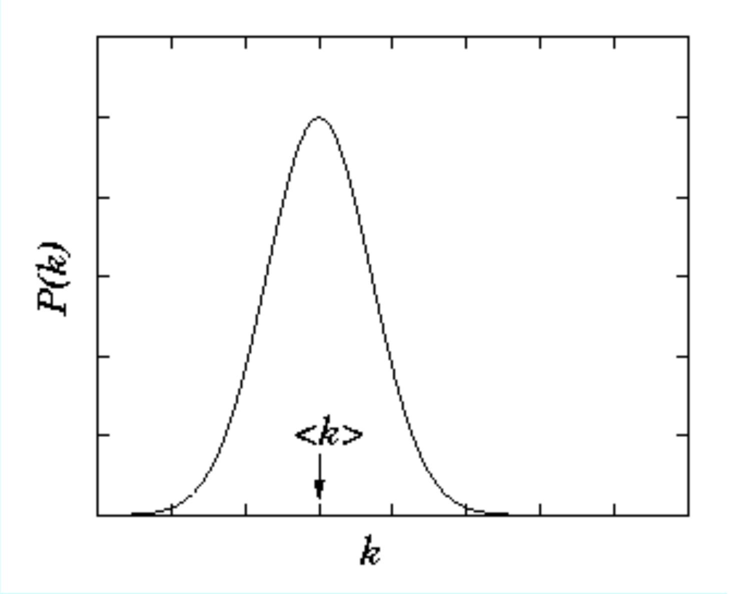




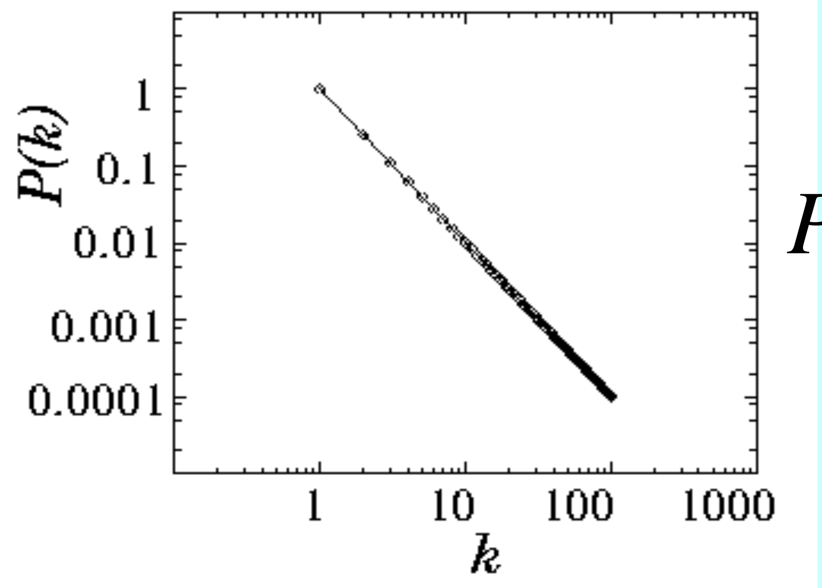
Scale-free networks

Barabasi-Albert
Rev. Mod. Phys 2001

Poisson distribution



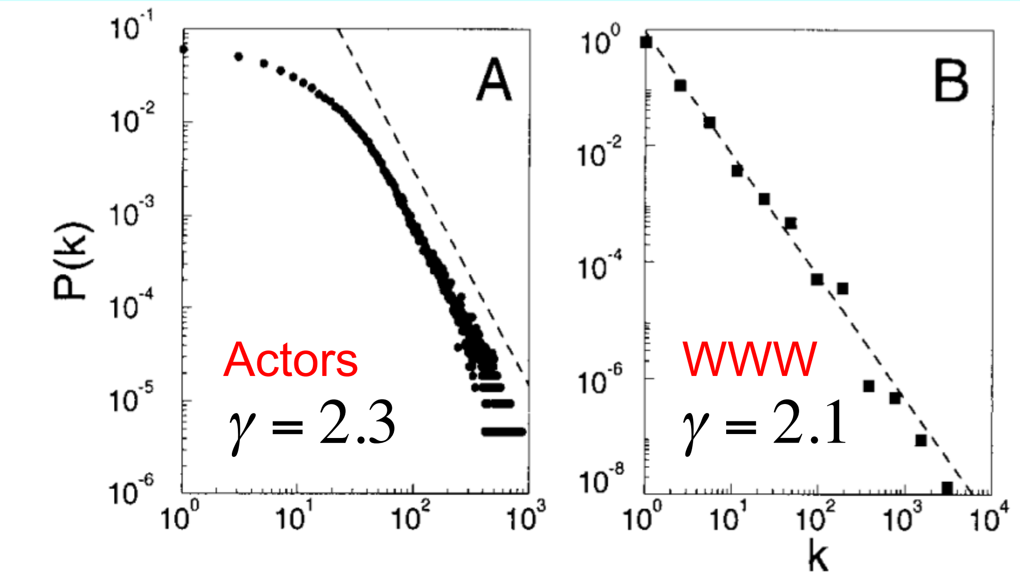
Power-law distribution



$$P(k) \approx k^{-\gamma}$$

Scale-free networks

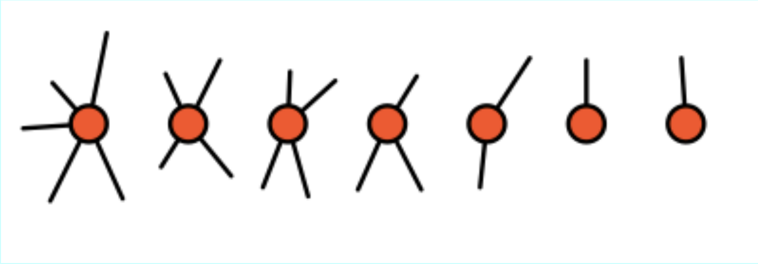
$$P(k) \approx k^{-\gamma}$$

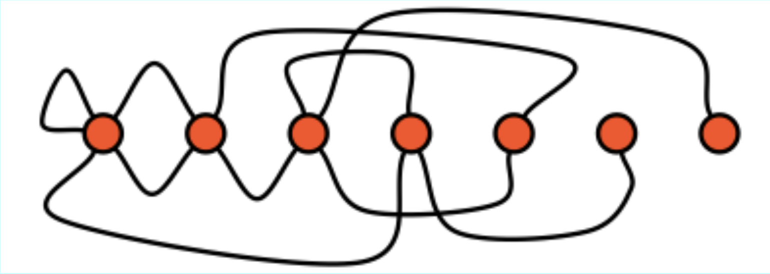


Networks	Exponent	
Internet	2.4	
WWW	2.1	2.7
Protein	2.4	
Metabolic	2.2	2.1
Coauthorship	2.5	
Movie actors	2.3	

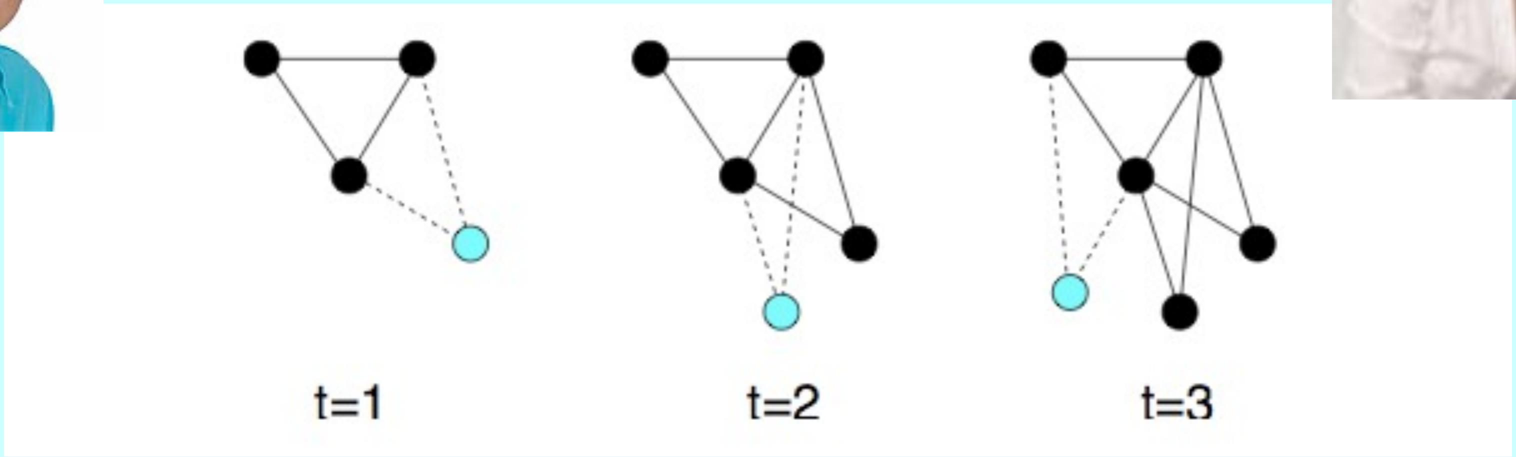
$$2 < \gamma \leq 3 \Rightarrow \begin{cases} \langle k \rangle \ finite \\ \langle k^2 \rangle \to \infty \end{cases}$$

Generalized random graphs





The Barabasi-Albert model



1) Growth : a node (with m links) is added at every time step

2) Linear preferential attachment: $\Pi_{n \rightarrow i} \propto k_i$

$$P(k) \xrightarrow[t \rightarrow \infty]{} c k^{-3}$$

Barabasi, Albert, Science 286, 509 (1999)

Epidemic spreading

SIS model

S = susceptible
I = infected

$$\begin{cases} S + I \xrightarrow{\lambda} I + I & \text{(Transmission rate)} \\ I \xrightarrow{\mu} S & \text{(Recovery rate)} \end{cases}$$

Transmission rate: λ

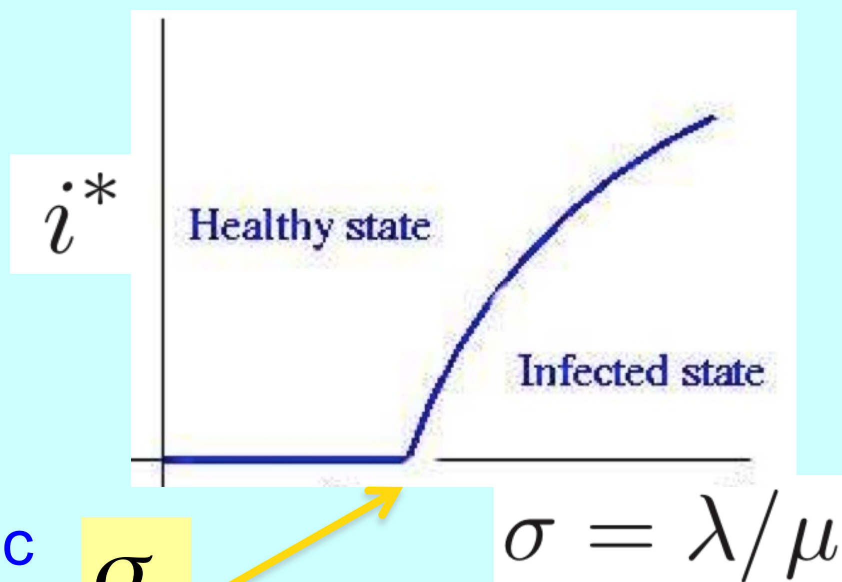
Recovery rate: μ

$$\sigma = \frac{\lambda}{\mu}$$

$$\frac{di(t)}{dt} = -\mu i(t) + \lambda \langle k \rangle i(t)[1 - i(t)]$$

No endemic state (i^*=0) if

$$\sigma < \sigma_c = \frac{1}{\langle k \rangle}$$



Epidemic threshold σ_c

Epidemic spreading in scale-free nets

Transmission rate

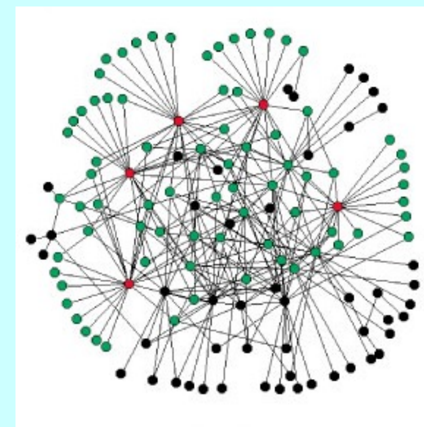
$$S + I \xrightarrow{\lambda} I + I$$

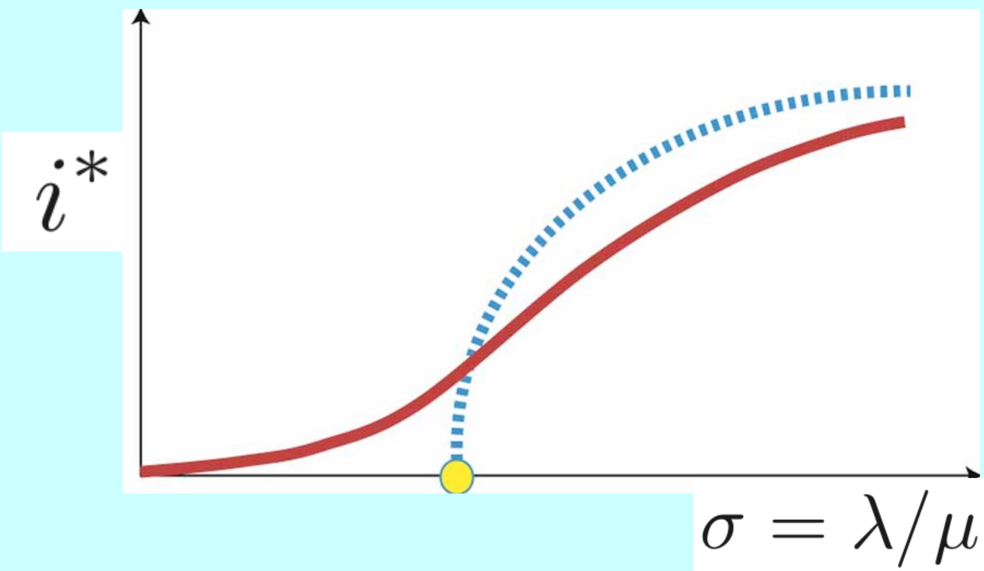
Recovery rate

$$I \xrightarrow{\mu} S$$

$$\frac{di_k(t)}{dt} = -i_k(t) + \lambda k \left[1 - i_k(t)\right] \Theta_k(t) \qquad k = 0, \ldots, N-1$$

$$\Theta_k(t) = \Theta[\{i_{k'}(t)_{k'=0,\ldots,N-1}\}] = \frac{1}{\langle k \rangle} \sum_{k'} k' P(k') i_{k'}(t)$$





$$\sigma_c = \frac{\langle k \rangle}{\langle k^2 \rangle} \xrightarrow[N \to \infty]{} 0$$

No epidemic threshold

Pastor-Satorras, Vespignani, PRL 86, 3200 (2001)

Structure and dynamics of complex nets

- Structural descriptors of networks from the real world

  node properties, degree distributions, degree-degree correlations, motifs, communities

- New graph models

  random graphs with P(k), models of graph growth, correlated graphs

- The structure affects the function

  percolation, diffusion, spreading of diseases, searching information, routing protocols, coupled dynamical systems

Albert, Barabasi, Rev. Mod. Phys. 74 (2002) 47
Newman, SIAM Rev. 45 (2003) 167
Boccaletti, Latora, Moreno, Chavez, Hwang, Phys Rep 424 (2006) 175

Complex Networks (MTH6142)

- 1) Introduction
- 2) Basic properties
- 3) Centrality measures
- 4) Random graphs
- 5) Scale-free networks
- 6) Growing networks
- 7) Small-world networks
- 8) The configuration model

Typed lecture notes (written by Prof Ginestra Bianconi in 2020/21) available from QMPlus.

- **Handwritten notes and slides:** Available on QMPlus after the lecture/tutorial

- **Useful textbooks**

  -Newman, Networks: An Introduction, Oxford University Press 2010

  -Barabasi Network Science Cambridge University Press 2016

  -Latora, Nicosia, Russo, Complex Networks Cambridge University Press 2017

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Networks constitute the backbone of complex systems, from the human brain to computer communications, transport infrastructures to online social systems, metabolic reactions to financial markets. Characterising their structure improves our understanding of the physical, biological, economic and social phenomena that shape our world.

Rigorous and thorough, this textbook presents a detailed overview of the new theory and methods of network science. Covering algorithms for graph exploration, node ranking and network generation, among the others, the book allows students to experiment with network models and real-world data sets, providing them with a deep understanding of the basics of network theory and its practical applications. Systems of growing complexity are examined in detail, challenging students to increase their level of skill. An engaging presentation of the important principles of network science makes this the perfect reference for researchers and undergraduate and graduate students in physics, mathematics, engineering, biology, neuroscience and social sciences.

VITO LATORA is Professor of Applied Mathematics and Chair of Complex Systems at Queen Mary University of London. Noted for his research in statistical physics and in complex networks, his current interests include time-varying and multiplex networks, and their applications to socio-economic systems and to the human brain.

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Principles, Methods and Applications

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VINCENZO NICOSIA
GIOVANNI RUSSO