MTH5123
Differential Equations
Introductory slides

Prof. Ginestra Bianconi
School of Mathematical Sciences, Queen Mary University of London

Queen Mary
University of London

What are ordinary differential equations?

An ordinary differential equation of order n for a function $y(t)$

is an equation of the form

$$\mathscr{F}\left(\frac{d^n y}{dt^n}, \frac{d^{n-1} y}{dt^{n-1}}, \ldots, \frac{dy}{dt}, y, t\right) = 0$$

(where the $n-th$ derivative $\frac{d^n y}{dt^n}$ occurs in $\mathscr{F}$)

The variable t is called the independent variable,

the variable y is called the dependent variable

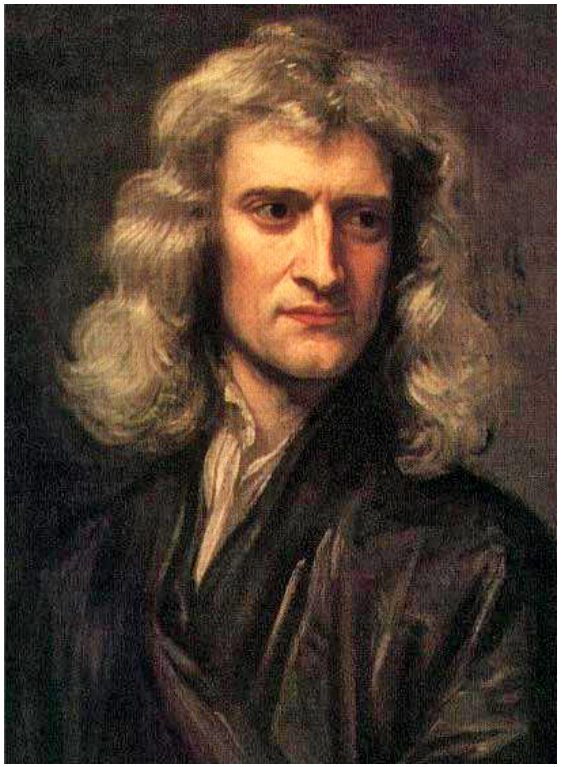
Isaac Newton
(25 December 1642 – 20 March 1726)

Driven by the need to understand

the physical word

including gravitation and mechanics

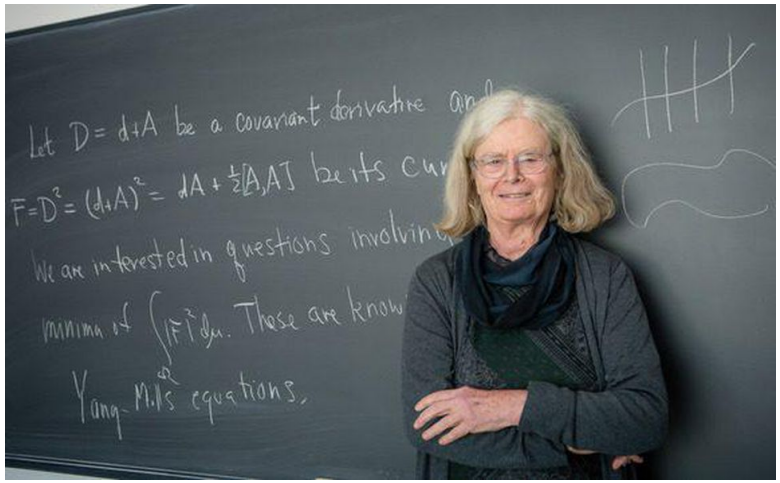
Isaac Newton founded Calculus



Notable mathematicians in differential equations

Katherine Johnson
(1918-2020)

Miryam Mirzakhani
(1977-2017)
Field Medal 2014

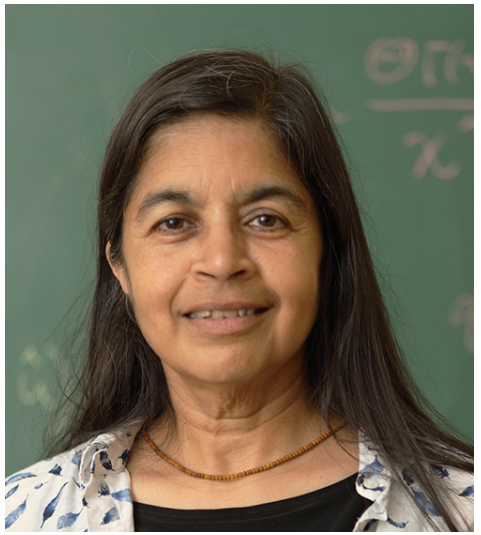


Karen Keskulla Uhlenbeck (born 1942) Abel Prize 2019

Notable scientists working with Differential equational and Dynamical Systems and applications

Syukuro Manabe Princeton University

Artur Avila Cordeiro de Melo University of Zurich



Nolini Joshi Sidney University

Classical mechanics: the birth of differential equations

At time t=0 you launch a mass vertically from a given height h at a given velocity v_0.

- Which is the height of the mass at time t?
- When does this mass reach the floor?
- At which velocity does it reach the floor?

$$m\ddot{y} = -mg$$

$$y(0) = h$$
$$\dot{y}(0) = v_0$$

Financial mathematics: Continuously Varying Interest Rates

Suppose you put x in your bank account at time $t = 0$, and the interest is continuously compounded with a rate $r(t)$ that changes in times.

- What is the amount $D(t)$ that you will have in your account at time t?

$$\dot{D}(t) = D(t)r(t)$$

$$D(0) = x$$

Modelling epidemic spreading

You want to model a pandemic

- What is the dynamics for a given R_0?
- How many would be the removed individuals (immunised or deceased) at time t?
- The SIR dynamics involving a given fraction of susceptible S, infected I and removed individuals R

$$S + I + R = 1$$

$$\dot{I} = R_0 I(1 - I - R) - I$$

$$\dot{R} = I$$

$$I(0) = I_0$$

$$R(0) = R_0$$

Climate change equation

- **Energy Balance models**

- Expresses how the temperature T of the Earth changes in time due to the balance between the radiation absorbed by the Sun S and the radiation emitted by the Earth $e\sigma T^4$, where the radiation emitted by Earth depends on the presence of Green House Gases in the atmosphere that change the emissivity e of the Earth.

$$c\rho\frac{dT}{dt} = (1-a)S - e\sigma T^4$$

This equation can be used to inform us in the implementation of the 2016 Paris Agreement to keep the Earth's temperature increase below 1.5 Degrees Centigrade

MTH5123: Applied mathematics module

The module intends to build
on your First and Second Year mathematical knowledge of

Calculus
Algebra

to give you the skills of being able to solve

Ordinary Differential equations

to address questions emerging in a variety of applications

Including physics, biology, finance etc.

You already know
how to solve some simple
differential equations of the form of

$$\frac{dy}{dx} = f(x)$$

such as $\frac{dy}{dx} = x$

having as solution

$$y(x) = \int x dx = \frac{x^2}{2} + C$$

Therefore it is important that you refresh your Calculus

In Week 1 please prepare

for the Differential Equations module

by

answering the

Revision questions of pre-request knowledge from previous modules

posted in the QM+ page under Week 1

Lectures and Tutorials

- Schedule:

- Lesson 1-2 Wednesday 11:00-13:00 Live Arts Two LT
- Lesson 3 Thursday 17:00-18:00 Live Arts Two LT

You will be assigned to one of the following three tutorials
(check your timetable to know which applies)

- Tutorial 1: Thursday 10:00-11:00 PP2
- Tutorial 2: Thursday 12:00-13:00 PP2
- Tutorial 3: Thursday 16:00-17:00 Graduate Ctr:GC201

Lectures and Tutorials

Your tasks:

- Make an effort to attend the lecture live
- Take your own notes while attending the lecture
- Ask questions
- Complete formative assessments, and mock quizzes and submit courseworks

Lecture notes

- Lecture note material available on QMPlus:
  - Typed in lecture notes
  - Handwritten lecture notes
- Your tasks:
  - Read the typed in lecture notes in advance of the lectures
  - Study the notes after the lecture to check your full understanding of the module material.

Reading list

- J. C. Robinson: An introduction to Ordinary Differential Equations (Cambridge University Press)

- Available from the library! (See link in QM+)

Good practice

Train yourself in deriving the solution

of differential equations with pen and pencil!!

This is of fundamental importance for the preparation to the final exam

A good mathematicians is like a good athlete:

he or she needs a lot of training and commitment!!

To check your solutions you can also help your study with numerical software such as Mathematica of MATLAB

See QM+ for an example of a Mathematica notebook

and link to QM student licence

Formative Assessment

- Each week, at the tutorial we will cover
  - The formative assignment and the mock quiz for the week, available on the QMPlus page
- Your task:
  - Attempt the formative assessment and the mock quiz before the tutorial
  - Ask questions regarding the formative assessment during the tutorial

Assessed courseworks

- You will have two assessed courseworks (quizzes).
- Every assessed coursework is worth 10% of the final marks
- You will have one week to complete assessed courseworks but once you open the quiz you need to complete it within 48 hours
- The 2 assessed courseworks will be posted in weeks 6,11

Final exam

- The final exam will account for 80% of the final mark
- Three questions on material week 1-6
- One question on material week 8-11

Feedback

Feedback on your assessed courseworks:

Personalised feedback

Quiz questions: You will be able to see your scores soon after the submission deadline

General feedback

General feedback will be given during the tutorial

where we will go over the most challenging questions of the assessed

courseworks and the common mistakes.

Support Learning Hour

Thursday 2:00-3:00pm

Online Forum

For any question on the module material that you would like to ask there are two ways to received feedback and answers:

- You can ask the question during the live lectures and tutorials
- You can post the question on the online forum

Participation to the online forum is highly beneficial as it increases the interactions between fellow students.

- The online forum will be monitored twice a week

Welcome to the Differential Equations module!

See you at the first lecture

11:00-13:00, Wednesday 27 September 2023