Queen Mary
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MTH5123 — Differential Equations

Formative Assessment Week 1 — G. Bianconi

- This Formative Assessment consists of three parts:
  - **I.** Practice problems. You will get help on this Formative Assessment in the tutorial of week 3. You should work on this before you go to this session.
  - **II.** Mock Quiz Week 1.
  - **III.** Exploration problems (to help you understand concepts discussed during lecture, not optional and examinable)
- A selection of solutions to the listed problems will be posted on QMPlus by the end of Week 1. You are expected to seek solutions to the remaining problems using the Reading List and making use of the tutorial sessions.
- I encourage all students to learn and check their computational answers using math softwares such as Mathematica, MATLAB, etc. Using numerical software is a fun practice and will help you to visualise your solutions.

I. Practice Problems

A. For each of the following differential equations compute the general solution by integration and fix the constant of integration according to the initial condition $y(0) = 1$.

1) $y' = 3x^2 + 2x + 3$

2) $y' = -2\sin(2x) + 2\cos(2x)$

3) $y' = (x+1)e^{-x^2-2x}$

4) $y' = e^{-x}\cos(x)$

5) $y' = 3/(1-x)$

B. Solve each of the following differential equations by separation of variables. Whenever possible write the general solution in explicit form. For each solution fix the constant of integration according to the given initial condition.

In the general solution of a first order ODE, you obtain y(x), where the value of y depends on the value of x. Also in the general solution, there is always a constant arbitrary C. The initial condition, for example $y(0) = -1$ means that when x=0, y = -1. Using this condition, you will specify what is the value of the arbitrary C in the general solution. You will learn more of this in the following weeks.

1) $y' = -x/y, \quad y(0) = -2.$

2) $y' = (y^2+1)/y, \quad y(0) = 1$

3) $y' = (1+y^2)e^x, \quad y(0) = -1$

4) $y' = ye^x - 2e^x + y - 2, \quad y(0) = 0$

C. For the following differential equation

$$y' = 2x + y - 5$$

use an appropriate substitution to reduce it to a separable form and hence apply the separation of variables method to determine the general solution $y(x)$ of the original differential equation.

II. Mock Quiz

Check your understanding by answering Mock Quiz Week 1.

III. Further Exploration: Modelling a Physical System

A. In our Week 1 lecture, we used Newton's Second Law of motion to deduce an equation describing the motion of a falling object of mass m as

$$m\frac{dv}{dt} = mg - \gamma v,$$

where g is the acceleration due to gravity, γ is a constant called the drag coefficient and $v = v(t)$ denotes the velocity of the object.

1) Complete the following sentence: A solution to the differential equation above is a [ ] whose graph is a curve in the [ ]-[ ] plane.

2) Assuming $m = 10$ and $\gamma = 2$, investigate the behaviour of the above differential equation without solving it. *Hint: if* $v = 40$, *then* $\frac{dv}{dt} = 1.8$. *This means that the slope of a solution curve* $v = v(t)$ *is positive (with value of the slope being* 1.8*) at any point where* $v = 40$. *Use this idea to draw a slope field for values* $40 \leq v \leq 60$.

3) Verify that the constant function $v(t) = 49$ solves the differential equation. This solution is known as an *equilibrium solution*, corresponding in this case to a balance between drag and gravity (sometimes called the *terminal velocity* of the object).

4) Finally, solve this differential equation using separation of variables and draw some solution curves, called *integral curves*, for various integration constants. How are these curves related to your slope field and the equilibrium solution?