Solutions for MTH5125 Late Summer Examination, 2023

A life insurer issued a special endowment insurance policy to a life aged 35, with term 5 years. The death benefit, payable at the end of the year of death, is equal to £1,000 plus the reserve that would have been held at the end of the year had the policyholder been alive. The maturity benefit (benefit paid at maturity if not dead) is £1,000. The premium is £200, payable annually in advance. The basis is AM92 ultimate life table and the interest is 4% per annum.

a) Find the reserve just before the payment of the 3rd premium by using the recursive relation between reserves.

b) Find the policy value at issuance using the same recursive method as above.

c) Discuss whether the answer at part b) is what you expected.

Solutions

a) From the AM92 table we get:

q_x	x
0.000689	35
0.000724	36
0.000765	37
0.000813	38
0.00087	39

$$\left({}_tV_{x:\overline{n}|} + P\right)(1+i) = q_{x+t}\left(1{,}000 + {}_{t+1}V_{x:\overline{n}|}\right) + p_{x+t}\,{}_{t+1}V_{x:\overline{n}|}$$

As the benefit at maturity is £1,000 then: ${}_5V_{35:\overline{10}|} = 1{,}000$

$$\left({}_tV_{x:\overline{n}|} + P\right)(1+i) = 1{,}000q_{x+t} + {}_{t+1}V_{x:\overline{n}|}$$

$$\begin{aligned} {}_4V_{35:\overline{n}|} &= \frac{1}{1.04} \times 1{,}000q_{39} - 200 + 1{,}000 \\ &= 820.0837 \end{aligned}$$

$$\begin{aligned} {}_3V_{35:\overline{n}|} &= \frac{1}{1.04} \times 1{,}000q_{38} - 200 + 820.0837 \\ &= 600.8654 \end{aligned}$$

$$\begin{aligned} {}_2V_{35:\overline{n}|} &= \frac{1}{1.04} \times 1{,}000q_{37} - 200 + 600.8654 \\ &= 401.601 \end{aligned}$$

b)

$$\begin{aligned} {}_1V_{35:\overline{n}|} &= \frac{1}{1.04} \times 1{,}000q_{36} - 200 + 401.601 \\ &= 202.2971 \end{aligned}$$

$$
\begin{aligned}
{}_0V_{35:\overline{n}|} &= \frac{1}{1.04} \times 1,000 q_{35} - 200 + 202.2971 \\
&= 2.9596
\end{aligned}
$$

c) At issuance one expects the EPV of the random loss (which is equal to ${}_0V$) to be zero. However the premium of £200 was not necessarily set under the same basis - for example the interest rate might have been different. In this case we would not expect this to hold.

2. A life assurance company issued a contingent assurance policy to the twin brothers, Alex and John. The policy provides £300,000 immediately on the death of Alex, if John is alive at that date. The brothers are exactly 50 years old when the policy is signed. The insurer treats the lives of John and Alex as independent. Annual premiums are payable in advance until the first death. Comission is 20% the premium in the first year and 5% of all premiums after the first year. We assume 4% per annum interest, and $\ddot{a}_{50:50} = 17.688$ and $\ddot{a}_{60:60} = 14.090$.

It is now ten years since the issue date; both brothers are still alive and the due premium for the current year has not yet been paid. The brothers have agreed to offer the policy for surrender to the insurer.

a) Calculate the gross annual premium that the brothers accepted to pay when they took the contingent policy.

b) Assuming that the life insurance company is prepared to give a surrender value equal to 95% of the gross premium reserve on the policy, calculate the surrender value that the insurer agrees to pay the brothers.

b) Before agreeing to give a surrender value, what should the life insurance company ascertain?

Solution

a) First we need to look at the equivalence principle of the initial policy:

$$
EPV(\text{benefit outgo+expenses}) = EPV(income)
$$

$$
\begin{aligned}
300,000 \times \bar{A}_{\overset{1}{50}:50} + 0.5P\ddot{a}_{50:50} + 0.15P &= P\ddot{a}_{50:50} \\
300,000 \times \bar{A}_{\overset{1}{50}:50} &= \left(0.95\ddot{a}_{50:50} - 0.15\right) P \\
P &= \frac{300,000 \times \bar{A}_{\overset{1}{50}:50}}{0.95\ddot{a}_{50:50} - 0.15}
\end{aligned}
$$

$\bar{A}_{\overset{1}{50}:50} = \frac{1}{2}\bar{A}_{50:50} = \frac{0.04}{2\ln(1.04)}(1 - d\,\ddot{a}_{50:50}) = 0.163022$

$P = 300,000 \times \frac{0.16302218}{0.95 \times 17.688 - 0.15} = 2936.7016$

b) Gross premium reserve:

$$\begin{aligned}{}_{10}V &= 300,000\ \bar{A}_{\overset{1}{60:60}} - P\ \ddot{a}_{60:60} \\ &= 300,000\ \bar{A}_{\overset{1}{60:60}} - P\ \ddot{a}_{60:60}\end{aligned}$$

Note that P is expense loaded premium (gross).
$\bar{A}_{\overset{1}{50:50}} = \frac{1}{2}\bar{A}_{50:50} = \frac{0.04}{2\ln(1.04)}(1 - d\ \ddot{a}_{50:50}) = 0.16302$
$\bar{A}_{\overset{1}{60:60}} = \frac{1}{2}\bar{A}_{60:60} = \frac{0.04}{2\ln(1.04)}(1 - d\ \ddot{a}_{60:60}) = 0.23359$
we have

$${}_{10}V = 28,698.66$$

Therefore the surrender value is 27,263.73

c) The insurer should ascertain the state of health of the twin brother and whether he is likely to engage in risky pursuits in the fairly near future since his death (before that of the other twin's death) would render the policy worthless.

3. Consider a fully discrete whole life insurance with sum insured £190,000 issued to a life aged 35. The level premiums are paid annually in advance and the payment term is 20 years. Commissions are 7% of the first premium and 5% of the subsequent premiums. Assume the mortality follows the AM92 ultimate life table with $i = 4\%$ per annum.

a) Write down an expression for the gross loss at issue random variable
b) Calculate the annual premium.
c) Calculate the probability that the contract makes a profit

Solutions
a) Gross future loss at issue:

$$L_0^n = 190,000v^{K_{35}+1} + 0.05\ddot{a}_{\overline{\min(K_{35}+1,20)}|} + 0.02P - P\ddot{a}_{\overline{\min(K_{35}+1,20)}|}$$

Equivalence principle:

$$E\left(L_0^n\right) = 190,000A_{35} - 0.95P\ddot{a}_{35:\overline{20}|} + 0.02P = 0$$

b)
$\ddot{a}_{35:\overline{20}|} = \ddot{a}_{35} - \ v^{20}\ {}_{20}p_{35}\ddot{a}_{55}$
${}_{20}p_{35} = \frac{l_{55}}{l_{35}} = \frac{9557.9179}{9894.4299} = 0.96699$
$\ddot{a}_{35:\overline{20}|} = 21.003 - (\frac{1}{1.04})^{20} \times 0.96699 \times 15.873 = 14.03215$

$$\begin{aligned}P &= \frac{190,000A_{35}}{0.95\ddot{a}_{35:\overline{20}|} - 0.02} \\ &= \frac{190,000 \times 0.19219}{0.95 \times 14.03215 - 0.02} \\ &= 2743.3972\end{aligned}$$

c) Did the policy made a profit at death?
At death:

- Benefit outgo: 190,000
- Income: premiums for 20 years only - bring them to the time of death?

$$\underbrace{\ddot{a}_{35:\overline{20}|} \times P\,(1.04)^{20} \times}_{FV \text{ at the end of 20 years}} (1.04)^{n-20} > 190,000$$

$$\ddot{a}_{35:\overline{20}|} \times P\,(1.04)^{20} \times (1.04)^{n-20} = \frac{190,000}{\ddot{a}_{35:\overline{20}|} \times P\,(1.04)^{20}}$$

$$\begin{aligned} n &= \left\lfloor \ln\left(\frac{190,000}{\ddot{a}_{35:\overline{20}|} \times P\,(1.04)^{20}}\right) / \ln(1.04) + 20 \right\rfloor \\ n &= 58 \end{aligned}$$

We want the smallest integer such that the accumulations of premiums to time n exceed the sum insured

$$n = 58$$

The life needs to live 57 years or more for profit to be made, hence probability of profit ${}_{57}p_{35} = 11.34\%$

4. A multiple life model for a family (Parents and Child) has 6 states:
State 0: Both parents are alive, child is alive.
State 1: One parent is dead, one parent is alive, child is alive.
State 2: Both parents are alive, child is dead.
State 3: Both parents are dead, child is alive.
State 4: One parent is alive, one parent is dead, child is dead.
State 5: Both parents are dead, child is dead.

You are given the following constant forces of transition: $\mu^{01} = \mu^{13} = \mu^{24} = \mu^{45} = 0.05$ $\mu^{02} = 0.01$ $\mu^{14} = \mu^{35} = 0.02$

A life assurance company offers a fully continuous whole life insurance policy for this family. The policy pays £100,000 when a parent dies and another £200,0000 when the other parent dies. No benefit will be paid for the child's death. The policy will automatically expires when both parents are dead OR when the child is dead. Premiums are payable continuously until the policy expires. The force of interest is $\delta = 0.10$.

a) Create a transition diagram for this model.

b)Express each of ${}_tp_x^{00}, {}_tp_x^{01}, {}_tp_x^{11}$ as a function of t.

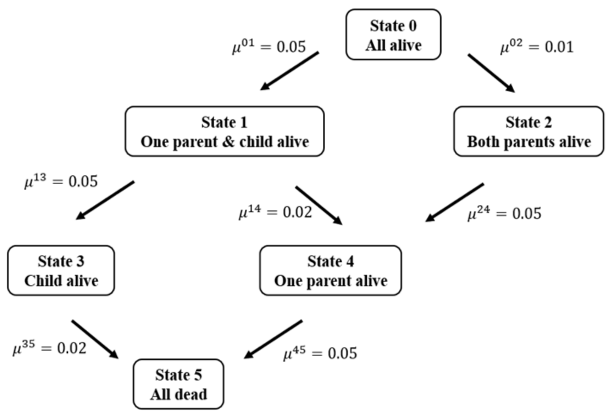
c) Calculate the expected present value of death benefits.

d) Calculate the net annual premium.

e) Describe the child's future lifetime if μ^{02} is increased to 0.02.

Solutions

a)



b)

$$ {}_tp_x^{00} = e^{-(0.05+0.01)t} = e^{-0.06t} $$

$$ {}_{t-s}p_{x+s}^{11} = e^{-(0.05+0.02)(t-s)} = e^{-0.07(t-s)} $$

$$ \begin{aligned} {}_tp_x^{01} &= \int_0^t {}_sp_x^{00}\, \mu_{x+s}^{01}\, {}_{t-s}p_{x+s}^{11}\, dt \\ &= \int_0^t e^{-0.06s} 0.05 e^{-0.07(t-s)}\, dt \\ &= 0.05e^{-0.07t} \int_0^t e^{0.01s}\, ds \\ &= 0.05e^{-0.07t} \frac{e^{0.01t}-1}{0.01} \\ &= 5(e^{-0.06t} - e^{-0.07t}) \end{aligned} $$

$\bar{A}_x^{01} = \int_0^\infty v^t {}_tp_x^{00}\, \mu_{x+t}^{01}\, dt =$

$\int_0^\infty e^{-0.10t} e^{-0.06t} 0.05\, dt = \frac{0.05}{0.16} = 0.3125$

$$\begin{aligned}\bar{A}_x^{03} &= \int_0^\infty v^t {}_tp_x^{01}\, \mu_{x+t}^{13}\, dt \\ &= \int_0^\infty e^{-0.10t} 5(e^{-0.06t} - e^{-0.07t})(0.05)\, dt \\ &= 5(0.05)\left(\frac{1}{0.16} - \frac{1}{0.17}\right) \\ &= 0.091912\end{aligned}$$

$EPV(B) = 100,000 \times 0.3125 + 200,000 \times 0.091912 = 49,632.35$

d)

$\bar{a}_x^{00} = \int_0^\infty v^t {}_tp_x^{00}\, dt =$

$$\int_0^\infty e^{-0.10t} e^{-0.06t}\, dt = \frac{1}{0.16} = 6.25$$

$\bar{a}_x^{01} = \int_0^\infty v^t {}_tp_x^{01}\, dt =$

$$\int_0^\infty e^{-0.10t} 5(e^{-0.06t} - e^{-0.07t})\, dt$$

$= 5\left(\frac{1}{0.16} - \frac{1}{0.17}\right) = 1.838235$

$EPV(P) = P(6.25 + 1.838235) = 8.088235P$
$E[L_0^n] = 49,632.35 - 8.088235P = 0$
$P = 6,136.363$

e) The child's future lifetime would be independent of his/her parents' future lifetimes. so nothing changes.

5. A life assurance company is planning to launch a new five year endowment assurance. There is some debate over the final product design so the following results have been produced for two options, A and B.

Option Profit Signature:

$A: \quad (-1,378 \quad 550 \quad 700 \quad 442 \quad 200)$
$B: \quad (-700 \quad 100 \quad 100 \quad 240 \quad 65)$

The insurer wishes to proceed with the product with the shortest discounted payback period, using a risk discount rate of 7% per annum.

a) Determine which design the insurer should choose.

b) Explain why the discounted payback period is a useful profit criterion for the insurer.

c) The design that you have identified in part a) will be sold to policyholders aged 35, for a premium of £2,000 payable annually in advance. Calculate the profit margin that the insurer will receive on the contract, given that the mortality follows the AM92 ultimate life table.

Solution

a) The insurer would choose contract A because it has a DPP of 3 years compared to contract B which has DPP of 4 years.

	v	0.934579439	
		discounted	cdf
A	-1,378	-1287.8505	-1287.8505
	550	480.3913	-807.4592
	700	571.4085	-236.0507
	442	337.1997	101.1490
	200	142.5972	243.7463
		243.7463	

	v	0.934579439	
		discounted	cdf
B	-700	-654.2056075	-654.2056075
	100	87.34387283	-566.8617346
	100	81.62978769	-485.231947
	240	183.0948509	-302.1370961
	65	46.34410167	-255.7929944
		-255.7929944	

b) Insurers only have limited amounts of capital. The DPP measures how quickly that capital is returned to the insurer so that they can invest in selling new policies.

c) From AM92 we get q_x :

q_x	x
0.000689	35
0.000724	36
0.000765	37
0.000813	38
0.00087	39

	v	0.934579439		
		discounted	cdf	q_x
A	-1,100	-1028.0374	-1028.0374	0.000689
	345	301.3364	-726.7010	0.000724
	500	408.1489	-318.5521	0.000765
	530	404.3345	85.7824	0.000813
	200	142.5972	228.3796	0.00087
		228.3796		
	Year	probability	dicount	premium
	1	1	1	2000
		0.999311	0.934579439	1867.871028
		0.998587	0.873438728	1744.409119
		0.997822	0.816297877	1629.03996
		0.997009	0.762895212	1521.226785
				8762.546892
			Profit margin	2.78%

IFoA mapping

Question 1: Pricing and reserving

- CM1 Syllabus Objective: 6.1
- Standard question (similar to class exercises)

Question 2: Joint life and Pricing and reserving

- Standard question (similar to class exercises) - last partmore difficult as it tests further understanding.
- CM1 Syllabus Objective: 5.1, 5.2

Question 3: Gross premium calculations, Pricing and reserving

- CM1 Syllabus Objective: 6.1, 6.2
- Part a) and b) standard questions (similar to class exercises). Part c) medium level of difficulty

Question 4: Multiple decrement and multiple life models

- Standard question (similar to class exercises) - last part more difficult as it tests further understanding.
- CM1 Syllabus Objective: 5.1, 5.2

Question 5: Pricing and reserving

- CM1 Syllabus Objective: 6.4
- Standard question

All guestions are unseen.