**Late-Summer Examination period 2023**

# MTH5125: Actuarial Mathematics II

**Duration: 2 hours**

The exam is intended to be completed within **2 hours**. However, you will have a period of **4 hours** to complete the exam and submit your solutions.

**For actuarial students only:** This module also counts towards IFoA exemptions. For your submission to be eligible, **you must submit within the first 3 hours.**

**You should attempt ALL questions. Marks available are shown next to the questions.**

All work should be **handwritten** and should **include your student number**. Only one attempt is allowed – **once you have submitted your work, it is final**.

In completing this assessment:

- You may use books and notes.
- You may use calculators and computers, but you must show your working for any calculations you do.
- You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.
- You must not seek or obtain help from anyone else.

When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;
- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;

**Examiners: M. Nica, L. Fang**

**Question 1 [19 marks].** A life insurer issued a special endowment insurance policy to a life aged 35, with term 5 years. The death benefit payable at the end of the year of death, is equal to £1,000 plus the reserve that would have been held at the end of the year had the policyholder been alive. The maturity benefit (benefit paid at maturity if the policyholder still alive) is £1,000. The premium is £200, payable annually in advance. The basis is AM92 ultimate life table and the interest is 4% per annum.

(a) Find the reserve just before the payment of the 3rd premium by using the recursive relation between reserves. **[7]**

(b) Find the policy value at issuance using the same recursive method as above. **[7]**

(c) Explain why your answer to (b) is not consistent with the equivalence principle and give two examples of how this may have arisen. **[5]**

**Question 2 [21 marks].** A life assurance company issued a contingent assurance policy to the twin brothers, Alex and John. The policy provides £300,000 immediately on the death of Alex, if John is alive at that date. The brothers are exactly 50 years old when the policy is signed. The insurer treats the lives of John and Alex as independent. Annual premiums are payable in advance until the first death. Comission is 20% of the premium in the first year and 5% of all premiums after the first year. We assume 4% per annum interest, and $\ddot{a}_{50:50} = 17.688$ and $\ddot{a}_{60:60} = 14.090$. It is now ten years since the issue date; both brothers are still alive and the due premium for the current year has not yet been paid. The brothers have agreed to offer the policy for surrender to the insurer.

(a) Calculate the gross annual premium that the brothers accepted to pay when they took the contingent policy. **[7]**

(b) Assuming that the life assurance company is prepared to give a surrender value equal to 95% of the gross premium reserve on the policy, calculate the surrender value that the insurer should expect to pay. **[7]**

(c) Before agreeing to give a surrender value, what should the life assurance company ascertain? **[7]**

**Question 3 [20 marks].** Consider a fully discrete whole life insurance with sum insured £190,000 issued to a life aged 35. The level premiums are paid annually in advance and the payment term is 20 years. Commissions are 7% of the first premium and 5% of the subsequent premiums. Assume the mortality follows the AM92 ultimate life table with $i = 4\%$ per annum.

(a) Write down an expression for the gross loss at issue random variable. **[5]**

(b) Calculate the gross annual premium. **[5]**

(c) Calculate the probability that the contract makes a profit. **[10]**

**Question 4 [23 marks].** A multiple life model for a family (two parents and one child) has 6 states:

State 0: Both parents are alive, child is alive.
State 1: One parent is dead, one parent is alive, child is alive.
State 2: Both parents are alive, child is dead.
State 3: Both parents are dead, child is alive.
State 4: One parent is alive, one parent is dead, child is dead.
State 5: Both parents are dead, child is dead.

You are given the following constant forces of transition:
$\mu^{01} = \mu^{13} = \mu^{24} = \mu^{45} = 0.05$, $\mu^{02} = 0.01$, $\mu^{14} = \mu^{35} = 0.02$.

A life assurance company offers a fully continuous whole life insurance policy for this family. The policy pays £100, 000 when a parent dies and another £200, 0000 when the other parent dies. No benefit will be paid for the child's death. The policy will automatically expire when both parents are dead or when the child is dead. Premiums are payable continuously until the policy expires. The force of interest is $\delta = 0.10$.

(a) Create a transition diagram for this model. **[5]**

(b) Express each of ${}_tp_x^{00}$, ${}_tp_x^{01}$, ${}_tp_x^{11}$ as a function of $t$. **[5]**

(c) Calculate the expected present value of death benefits. **[5]**

(d) Calculate the net annual premium. **[5]**

(e) Describe the child's future lifetime if $\mu^{02}$ is increased to 0.02. **[3]**

**Question 5 [17 marks].** A life assurance company is planning to launch a new five year endowment assurance. There is some debate over the final product design so the following results have been produced for two options, $A$ and $B$. Option Profit Signature (in pounds):

$$\begin{array}{ll} A: & (-1,378 \quad 550 \quad 700 \quad 442 \quad 200) \\ B: & (-700 \quad 100 \quad 100 \quad 240 \quad 65) \end{array}$$

The insurer wishes to proceed with the product with the shortest discounted payback period, using a risk discount rate of 7% per annum.

(a) Determine which design the insurer should choose. **[7]**

(b) Explain why the discounted payback period is a useful profit criterion for the insurer. **[3]**

(c) The design that you have identified in part a) will be sold to policyholders aged 35, for a premium of £2, 000 payable annually in advance. Calculate the profit margin that the insurer will receive on the contract, given that the mortality follows the AM92 ultimate life table. **[7]**

---

**End of Paper.**