

MTH6113 Specimen Paper 3 – Total Time 3 hours

1. An investor has a utility from wealth described by: $U(w) = \sqrt{w}$ where $w > 0$ is his/her wealth.
- a) Show that this investor prefers more to less and he/she is risk averse. [4 marks]
 - b) What are this investor's Arrow Pratt measures of risk aversion? Are these measures decreasing or increasing in wealth? Explain what these results convey. [4 marks]
 - c) Initially the investor has a wealth of £400. He also has a lottery ticket that will be worth £1,200 with probability 0.5 and £0 with probability 0.5. What is his/her expected utility? [2 marks]
 - d) Someone offers to buy this lottery ticket. What is the lowest price at which the investor will part with the lottery ticket? [4 marks]
 - e) The investor considers investing in the two risky assets A and B with cumulative probability distribution functions:

$$F_A(w) = w$$

$$F_B(w) = w^{1/3} + w$$

- i) Find the conditions for which these cdfs are well defined. [2 marks]
- ii) Under what conditions on w is A preferred to B on the basis of first-order stochastic dominance? [2 marks]
- iii) Under what conditions on w is A preferred to B on the basis of second-order stochastic dominance? [4 marks]

Total: [22 marks]

2. An investor has access only to two stocks General Motors (GM) and Fiat Chrysler Automobiles (FCAU). The expected annual return on GM's shares is 10%, and the expected annual return on FCAU's shares is 7%. These stocks trade on NYSE. Today's price per share of GM opened at \$43.40 and of FCAU at \$24.50. The rates of return from these two companies' shares have a correlation coefficient of 0.8. The standard deviation of the rates of return on GM's shares is 0.04 and the standard deviation of the return on FCAU's shares is 0.08. The investor prefers more to less and can short sell both assets.

- a. The investor buys 100 shares of GM and 10 shares of FCAU at the open price.
 - i) What is the market price of this portfolio? [2 marks]
 - ii) What are the weights of GM and FCAU in this portfolio? [2 marks]
- b. What is the minimum global variance portfolio (V) for this investor? Interpret the weights on GM and FCAU. [4 marks]
- c. What is the expected value and variance of the portfolio found at point (c)? [4 marks]

- d. Draw the Mean Variance Frontier for this investor. Make sure you identify on the diagram the GM and FCAU stocks as well as the minimum variance portfolio V. Based on your diagram can you invest only on one of the stocks efficiently?

[4 marks]

- e. Assume now that GM and FCAU are perfectly positively correlated. Can you offer a security with no risk in this economy? If yes, what are the weights of GM and FCAU in this new security?

[4 marks]

Total: [20 marks]

3. An investor believes that the upcoming referendum on the independence of one of the regions of the country results will play a role in the success of his business. There are only two possibilities, the region becomes independent or not. Recent polls have predicted that the electorate is split at 50% between the options.

The investor believes that its venture percentage annual returns, R , in the post-referendum scenario will have the following distribution: $R \sim \text{Uniform}(-2, 2)$ if the L Party wins and $R \sim \text{Uniform}(-2, 6)$ if the R Party wins. Note that annual return is a continuous random variable.

- a) Calculate the conditional expected returns and variances of returns for the company, separately for the two possible alternative post-election scenarios.

[4 marks]

- b) Calculate the unconditional expected returns and unconditional variance of returns for the company.

[6 marks]

- c) Calculate the probability that the company will register negative returns.

[4 marks]

- d) Calculate the Value-at-Risk over a period of one year for the investment return R for the following probabilities of ruin equal to 0.375 and 0.1.

[4 marks]

- e) Based on the different measures of risks and returns calculated above, summarise the company's outlook in the year following the election.

[4 marks]

[Total: 22 marks]

4. Suppose the annual rate of return on short-term government securities (risk-free) is 3%. Suppose asset A has a beta of 2 and an expected annual return of 15%.

- a) What is the expected return on the market according to CAPM

[4 marks]

- b) Draw a diagram showing the security market line, the risk free rate, the expected annual return of the market and the annual return of asset A.

[4 marks]

- c) Calculate the expected annual return on an asset B with a beta of 0.7

[4 marks]

- d) Suppose you bought asset B at £10 and sold it after one year for £12. Calculate the realized annual return on asset B.

[4 marks]

e) Determine whether asset B is overpriced or underpriced by the market.

[4 marks]

f) Explain the difference between security market line and capital market line.

[4 marks]

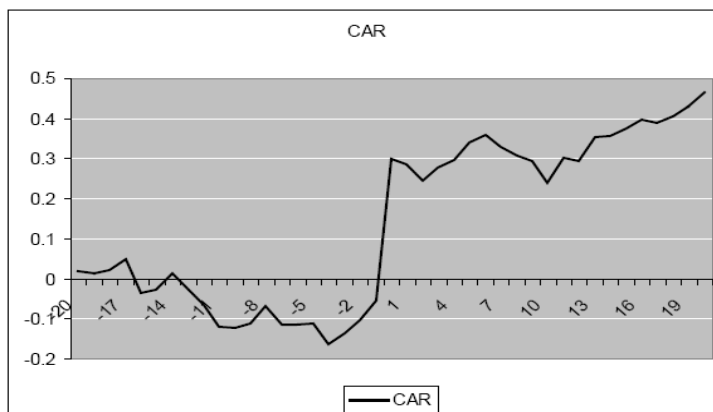
Total: [24 marks]

5.

a) The market of a small developing nation is currently semi-strong form efficient. The government announces that, in order to make the market strong form efficient, it is passing a law forbidding employees of companies from transacting in shares of their own companies. Defining semi-strong and strong forms of efficient market hypothesis, discuss if the proposed law would succeed.

[6 marks]

b) On May 20, 2024, after the close of trading, MEE (Massey Energy, a mining company) announced its third quarter earnings. On May 21st the stock reported a return of 37%. The graph below represents the daily Cumulative Abnormal Return for MEE for a 41-trading day window centred around the announcement day (day 0). Is the evidence in this event study consistent with the efficient market hypothesis?



[6 marks]

[Total: 12 marks]