Mathematical Tools for Asset management MTH6113

Revision

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Topics

- Review of Statistics
- The Basis of Economic Theory: Utility Theory
  - Concepts and Optimisation
  - Stochastic Dominance
- Portfolio Theory
  - Understanding Risk and Measures of Risks
  - Mean-Variance Analysis
- Assets Pricing
  - Capital Asset Pricing Model (CAPM)
  - Factor Models and Arbitrage Pricing Theory (APT).
- Efficient Market Hypothesis
- Log-normal Model for stock prices
- Behavioral Finance

Consumer Optimization Problem

Consumer' s Preferences
Indifference curve: a set of bundles among which the consumer is indifferent

Assumptions on consumers' preferences:

1. *Completeness*: either $X \succ Y$ or $Y \succ X$ or $X \sim Y$
2. *Transitivity*: $X \succ Y \succ Z$ then $Z \not\succ X$
3. *More-Is-Better*
4. Consumer preferences exhibit *diminishing marginal rates of substitution*.

Consumer Optimization Problem

Assuming we know the utility function of the consumer

Any agent's **decision problem**:

$$\max u(x, y)$$

- such that the budget constraint is satisfied:

$$p_x x + p_y y \leq M$$

- p_x- price of x
- p_y- price of y
- M- total available income

Consumer Optimization Problem

- Optimisation problem with inequality constraint: Langange method.
- Make sure you verify FOC and SOC

Expected Utility Theory

Generalise utility theory to consider situations that involve **uncertainty**

Any **risky asset** is characterised by a set of objectively known probabilities defined on a set of possible outcomes
The expected utility of a risky asset:

$$E\left[U\left(W\right)\right] = \sum_{i=1}^{N} p_i u\left(w_i\right)$$

When uncertainty present it is impossible to maximise utility with complete certainty

Maximise the expected value of utility given investor's particular beliefs about the probability of different outcomes

We still need to know the utility function and the probability distribution of the returns.

Expected Utility Theory and Risk

Risk aversion

A risk averse investor will reject a fair gamble

The utility function of a risk averse investor is a strictly concave function of wealth

Risk seeking

A risk seeking investor will seek a fair gamble

The utility function of a risk seeking investor is a strictly convex function of wealth

Risk neutrality

A risk neutral investor is indifferent to weather to accept or not a fair gamble

Expected Utility Theory and Risk

The certainty equivalent c_x **of a gamble** that provides an uncertain outcome x is determined by

$$E\left(U\left(w+x\right)\right) = U\left(w-c_x\right)$$

If the gamble is fair then a risk averse investor will reject a fair gamble i.e. keep their current wealth

$$E\left(U\left(w+x\right)\right) = U\left(w-c_x\right) < U\left(w\right)$$

The investor pays c_x to avoid the gamble (or has to be paid to take the gamble)

The principal underlying insurance

Absolute risk aversion

- The investor exhibits decreasing (increasing) absolute risk aversion (ARA) if $|c_x|$ decreases (increases) as wealth increases
  - Decreasing ARA: as wealth increases the absolute amount of wealth in risky assets increases

Relative risk aversion

- The investor exhibits decreasing (increasing) relative risk aversion (RRA) if $|\frac{c_x}{w}|$ decreases (increases) as wealth increases
  - Decreasing RRA: as wealth increases the relative amount of wealth in risky assets increases

Arrow-Pratt measures of Risk Aversion

Absolute Risk Aversion

$$A(w) = -\frac{U''(w)}{U'(w)}$$

Relative Risk Aversion

$$R(w) = -w\frac{U''(w)}{U'(w)}$$

Stochastic Dominance

Question: How can we rank investments/gambles/lotteries if

- **we don't know the exact individual utility function**
- **but we know the distribution of returns of investment**

Second Order Stochastic Dominance

Portfolio A **first order dominates** B (the investor will prefer Portfolio A to Portfolio B) if:

$$\begin{aligned} F_A(x) &\leq F_B(x) \text{ for all } x \text{ and} \\ F_A(x) &< F_B(x) \text{ for some } x \end{aligned}$$

Portfolio A **second order dominates** B if:

$$\begin{aligned} \int_a^x F_A(y)\,dy &\leq \int_a^x F_B(y)\,dy \text{ for all } x \text{ and} \\ \int_a^x F_A(y)\,dy &< \int_a^x F_B(y)\,dy \text{ for some } x \end{aligned}$$

Measures of Investment Risk

Question: How can we rank investments/gambles/lotteries if

- if **we don't know the whole distribution of returns of investment/asset**

Answer: Use **partial known information on the distribution of returns** (i.e. moments of distribution)

What the industry uses?

Measures of Investment Risk

- Expected Value/Mean

$$E(X) \equiv \mu = \sum_i p_i x_i \text{ if } X \text{ is discrete}$$

$$E(X) \equiv \mu = \int_{-\infty}^{\infty} x f(x) dx \text{ if } X \text{ is continuous}$$

- Variance of return:

$$Var(X) \equiv \sigma^2 = \sum_i (x_i - \mu)^2 p_i \text{ if } X \text{ is discrete}$$

$$Var(X) \equiv \sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx \text{ if } X \text{ is continuous}$$

Measures of Investment Risk

- Downside semi-variance of return

$$SV(X) = \sum_{x_i \leq \mu} (x_i - \mu)^2 p_i \text{ if } X \text{ is discrete}$$

$$SV(X) = \int_{-\infty}^{\mu} (x - \mu)^2 f(x) dx \text{ if } X \text{ is continuous}$$

- Shortfall probabilities:

$$\sum_{p_i < L} p_i \text{ if } X \text{ is discrete}$$

$$\int_{-\infty}^{L} f(x) dx \text{ if } X \text{ is continuous}$$

Measures of Investment Risk

Value at Risk (VaR):

- the largest number L such that the probability that the loss on the portfolio is greater than VaR, is q
- relates to Shortfall Probability but specifies a probability q and calculates the corresponding shortfall

If X is discrete:

$$VaR\,(X;q) = -L \text{ where } L = \{\max x_i : \Pr\,(X < x_i) \le q\}$$

If X is continuous:

$$VaR\,(X;q) = -L \text{ where } L = \{\max x_i : \Pr\,(X < x_i) = q\}$$

or

$$VaR\,(X;q) = -L \text{ where } \Pr\,(X < L) = q$$

Value at Risk

- *VaR* is the mirror image of *SP*
- rather than specify a threshold value L and measure the probability, *VaR* specifies the probability and measures the corresponding threshold value
- *VaR* can be calculated from the probability of gains/losses during a period T
- *VaR* says: **We are** $100 - q$ **certain that we will not loose more than** £L **in time** T

Expected Shortfall

- *VaR* asks the question:
  - How bad things can go?
- Expected shortfall asks the question:
  - If things go bad, what is the expected loss?

Expected Shortfall

For a shortfall probability q and corresponding threshold L such that $\Pr(X < L) = q$ then expected shortfall is:

$$E\left[\max\left(L - X, 0\right)\right] = \sum_{x_i \leq L} \left(L - x_i\right) p_i \text{ for } X \text{ discrete}$$

$$E\left[\max\left(L - X, 0\right)\right] = \int_{-\infty}^{L} \left(L - x\right) f\left(x\right) dx \text{ for } X \text{ continuous}$$

For the $(1 - q) \times 100\%$ confidence limit,expected shortfall represents the expected loss in excess of the $q - th$ lower tail value.

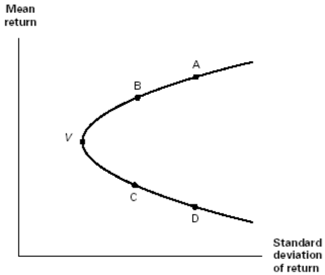
Mean Variance Portfolio Theory

Question: What is the optimal portfolio

Answer: Use **partial known information on the distribution of returns** (i.e. moments of distribution)

MVPT :The decision is based on **mean** and **variance of investment returns** over a single time horizon **given investor's preference for risk**

Mean-variance frontier is the expected return-variance locus:



Minimum variance portfolio: point V on the frontier

Optimal portfolio lies on the frontier and to the right of V

- investor's preferences are increasing in expected return and decreasing standard deviation

Important point: people could have different preferences between risk and return, so they might choose different locations on the frontier.

- B on the diagram represents and investor more risk averse than the investor located at A

Mean Variance Portfolio Theory

Investment Decision is:

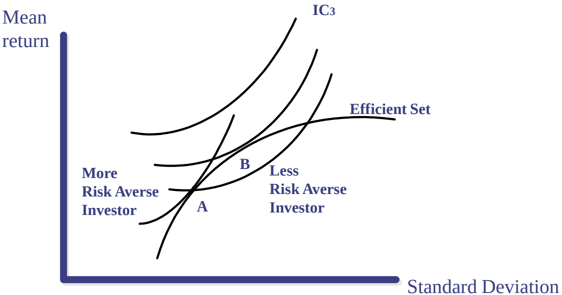
$$\min_{w_i} Var\left(R_P\right) \text{ such that}$$

$$\begin{aligned} E\left(R_P\right) &= E_P \\ \sum_{i=1}^{N} w_i &= 1 \end{aligned}$$

Constrained optimisation: use Lagrangian method!

For two security case **the global minimum variance** (point V on the diagram) occurs when:

$$w_1 = \frac{V_2 - C_{12}}{V_1 - 2C_{12} + V_2}$$

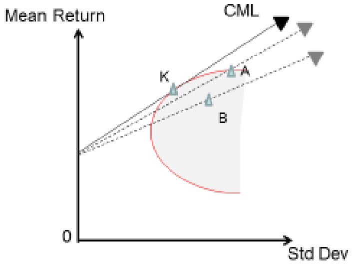


Capital Market Equilibrium: *demand for risky securities* $=$ *supply of risky securities*

- Demand for risky securities represented by the **tangency portfolio**
- Supply of risky assets summarized in **market portfolio**
  - market portfolio is the portfolio of all assets.
  - portfolio weights: market capitalization of each asset divided by the sum of market capitalization across all assets

Capital Market Line for an asset i **(CML$_i$)**: the slope of a portfolio consisting of a risk free asset and risky asset i

$$E(R_P) = r + [E(R_i) - r]\frac{\sigma_P}{\sigma_i}$$



Optimal Efficient Portfolio with risk free asset

Tangency Portfolio:

$$E\left(R_P\right) = r + \left[E_K - r\right] \frac{\sigma_P}{\sigma_K}$$

E_K: expected return of tangency portfolio

Capital Asset Pricing Model (CAPM)

Capital Market Equilibrium: *demand for risky securities = supply of risky securities*

Thus: **in equilibrium market portfolio and tangency portfolio are identical**

Capital Market Line - *the line denoting* **the efficient frontier** *in the CAPM model:*

$$E\left(R_P\right) - r = \left(E_M - r\right)\frac{\sigma_P}{\sigma_M}$$

$E\left(R_P\right)$: expected return of **any portfolio on efficient frontier**

It contains only efficient portfolios
$\frac{E_M - r}{\sigma_M}$: known as the market price of risk

Capital Asset Pricing Model (CAPM)

CAPM: relation between expected return on **any** asset j and the return on the market

Security Market Line (for any security):

$$E(R_j) = r + \beta_j (E(R_M) - r)$$

$$E(R_j) - r = (E_M - r)\frac{Cov(R_j, R_M)}{Var(R_M)}$$

$E_M - r$: **Market Risk Premium**

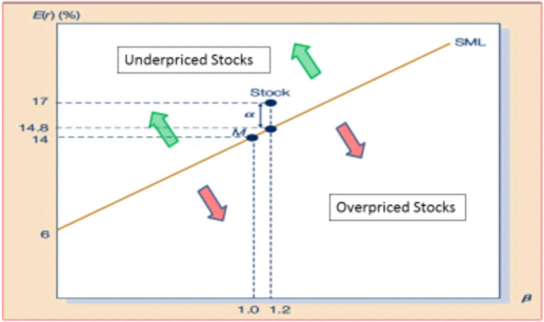
$\beta_j = \frac{Cov(R_j, R_M)}{Var(R_M)}$

What does “beta” mean?

- how the stock return covaries with the market portfolio return
- $\beta_j = 1$
- $\beta_j > 1$
- $\beta_j < 1$

Capital Asset Pricing Model (CAPM)

SML works as a benchmark to assess fair expected return on a risky asset



Alpha of a stock
α = actual expected return – required return (given risk) from CAPM

Capital Asset Pricing Model (CAPM)

- Market portfolio not observable
- Empirical test use broad-based equity index such FTSE-100, S&P500, Nikkei 250 as proxies

$$R_i = r + \beta_i \left(R_{index} - r \right) + \varepsilon_i$$

  - empirical studies do not strongly support the model
- The true market portfolio might contain other financial assets such as bonds and stock that are not in included in this indices as well as non financial assets: (real-estate, human capital).

Multifactor Models

- Assume returns are influenced not only by *market movements* but also by *other factors*
  - systematic risks (which cannot be diversified away)
- Multifactor Models:

$$R_i = \alpha_i + \beta_{i,1} I_1 + \beta_{i,2} I_2 + ... + \beta_{i,L} I_L + \varepsilon_i$$

$$\begin{aligned} E\left(\varepsilon_i\right) &= 0 \text{ for any } i \\ Cov\left(\varepsilon_i, \varepsilon_j\right) &= 0 \text{ for any } i, j \\ Cov\left(\varepsilon_i, I_l\right) &= 0 \text{ for any } i, l \\ E\left(I_l\right) &= 0 \text{ for any } l \end{aligned}$$

Multifactor Models

- I_l random variable
- I_l factors capture the variation of R_i about the expected return
- $E(R_i) = \alpha_i$
- Factors should proxy for risks and may be identified from economic fundamentals or empirical observations

Multifactor Models

- Macroeconomic Factor Models
  - use observable economic time series as factors
  - e.g. inflation and growth rates, interest rates, yields
  - could use a market index plus industry indices
- Fundamental Factor Models
  - use (possibly in addition) company specific variables
  - level of gearing and price earning ratio.
  - level of research & development spending.
- Statistical Factor Models
  - technique: Principal Components Analysis to determine a set of factors
  - in practice these factors rarely have a meaningful economic interpretation

Factor Replicating Portfolios

Definition: **A factor-replicating portfolio or pure factor portfolio** is a portfolio with unit exposure to one factor and zero exposure to others

Could I replicate a risk free asset?
Risk free asset replicating portfolio has a sensitivity to all factors I_j equal to 0.
Factor risk premium: $\lambda_j = E\left(R_{I_j}\right) - r$

Tracking Portfolios and Arbitrage Pricing Theory

Asset X tracking portfolio or asset X replicating portfolio

- build a security or portfolio made of assets X, Z, Y which tracks exactly asset X
- weight β_{jX} on the $j-th$ factor replicating portfolio R_{I_j} for any j
- weight $1-\sum_j \beta_{jX}$ on the risk free asset.
- The expected return on the X replicating portfolio (for two factor model) is:

$$
\begin{aligned}
E\left(R_X\right) &= \beta_{1X}\left(r+\lambda_1\right)+\beta_{2X}\left(r+\lambda_2\right)+\left(1-\beta_{1X}-\beta_{2X}\right)r \\
E\left(R_X\right) &= r+\beta_{1X}\lambda_1+\beta_{2X}\lambda_2
\end{aligned}
$$

- Remember the return on pure factor portfolio is $E\left(R_{Ij}\right)=r+\lambda_j$

Tracking Portfolios and Arbitrage Pricing Theory

Tracking Portfolios form the basis for Arbitrage Pricing Theory

- In the absence of arbitrage we require **all assets with identical factor exposures to earn the same return**
- **APT**: All securities and portfolios have expected returns described by:

$$E\left(R_i\right) = r + \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 + ... + \beta_{iL}\lambda_L$$

λ_l the risk associated with the lth factor

Efficient Market Theory

Jensen (1978):
A market is efficient with respect to a given information set Ω if no agent can make economic profit through the use of a trading rule based on Ω.

- economic profit: the level of return after costs are adjusted appropriately for risk

Efficient Market Hypothesis (EMH)**:** stock prices already reflect all available information, hence:

- **changes in prices** should be unpredictable (random)

Efficient Market Theory

Fama (1991):
EMH has three different versions based on definition of "all available information:"

- **Weak-form hypothesis**: stock prices already reflect all information that can be derived by examining *market trading data* such as historical prices, volumes, etc.

- **Semi-strong form hypothesis**: stock prices. incorporate *all publicly available information* regarding the firm's prospects (*market trading data* + future projects, earning forecasts)

- **Strong form hypothesis:** stock prices reflect *all information relevant to the firm* (including private information of company insiders + *all publicly available information*)

- If market is not weakly efficient, then it is not semi-strong efficient then it is not strong efficient.

Moving From Discrete Time to Continuous Time

- **Brownian motion** is a random walk occurring in continuous time
  - *with movements that are continuous rather than discrete*.
- A random walk can be generated by flipping a coin each period and moving one step
  - *with direction determined by whether the coin is heads or tails*.
- To generate Brownian motion, we would flip the coins infinitely fast and take infinitesimally small steps at each point.

Standard Brownian Motion

Definition

Standard Brownian Motion, SBM, is a stochastic process $\{B_t : t \geq 0\}$, with state space $S = \mathbb{R}$ (set of real numbers) and the following defining properties:$B_0 = 0$

- Definition

    1. $B_0 = 0$
    2. Independent increments: $B_t - B_s$ is independent of $\{B_r : r \leq s\}$, where $s < t$
    3. Stationary increments: Distribution of $B_t - B_s$ depends only on $(t - s)$, where $s < t$;the change in the value of the process over any two non- overlapping periods are statistically independent
    4. Gaussian increments: $B_t - B_s \sim N(0, t - s)$
    5. Continuity: B_t has continuous sample paths

Brownian Motion

Definition

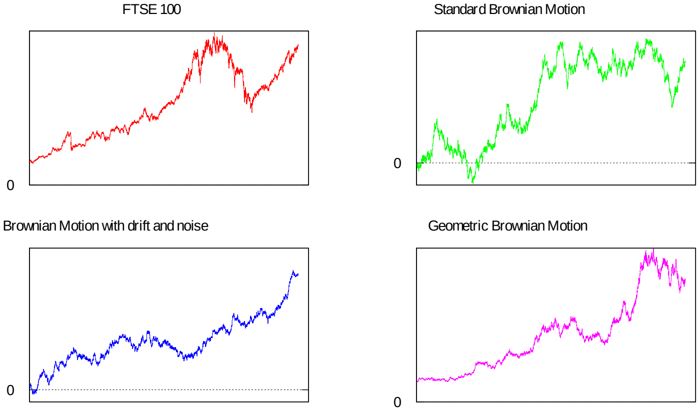
Brownian Motion, BM, is a stochastic process W_t, with state space $S = R$ (set of real numbers) and the following defining properties:

1. Independent increments: $W_t - W_s$ is independent of $\{W_r : r \leq s\}$, where $s < t$.
2. Stationary increments: Distribution of $W_t - W_s$ depends only on $(t - s)$, where $s < t$.
3. Gaussian increments: $W_t - W_s \sim N(\mu(t-s), \sigma^2(t-s))$.
4. Continuity: W_t has continuous sample paths.

Relationship between SBM and BM

- W_t (BM) can be obtained from B_t (SBM) by $W_t = W_0 + \mu t + \sigma B_t$
- μ - drift parameter and σ- volatility
- SBM can be obtained from BM by setting $\mu = 0$, $\sigma = 1$ and $W_0 = 0$.
- A **Geometric Brownian Motion** (GBM) is $S_t = \exp\left(W_t\right) = S_0 \exp\left(\mu t + \sigma B_t\right)$

Modelling Stock Prices



Geometric Brownian Motion (GBM) revisited

Consider the stock S_t with the stochastic differential equation:

$$dS_t = \alpha S_t dt + \sigma S_t dB_t$$

I want to find and expression for S_t
Standard Brownian Motion is nowhere differentiable despite the fact that it is continuous everywhere

- SBM is not a smooth function
- Can I use stochastic calculus to find an explicit formula for S_t?

Ito's Lemma

Ito's Lemma Let X_t be a stochastic process satisfying $dX_t = Y_t dB_t + Z_t dt$ and let $f(t, X_t)$ be a real-valued function, twice partially differentiable with respect to x and once with respect to t. Then $f(t, X_t)$ is also a stochastic process and is given by:

$$df(t, X_t) = \frac{\partial f}{\partial X_t} Y_t dB_t + \left[\frac{\partial f}{\partial t} + \frac{\partial f}{\partial X_t} Z_t + \frac{1}{2} \frac{\partial^2 f}{\partial X_t^2} Y_t^2 \right] dt.$$

Applying Ito's lemma to $f\left(t, S_t\right) = \ln S_t$:

Geometric Brownian Motion (GBM) revisited

Applying Ito's lemma to $f(t, S_t) = \ln S_t$:
Let $Y_t = \sigma S_t$ and $Z_t = \alpha S_t$

$$\begin{aligned} d \ln S_t &= \frac{1}{S_t}\sigma S_t dB_t + \left[0 + \frac{1}{S_t}\alpha S_t + \frac{1}{2}\left(-\frac{1}{S_t^2}\right)\sigma^2 S_t^2\right] dt \\ &= \left(\alpha - \frac{1}{2}\sigma^2\right) dt + \sigma dB_t \end{aligned}$$

Geometric Brownian Motion (GBM) revisited

$$S_t = S_0 \exp\left[\left(\alpha - \frac{1}{2}\sigma^2\right)t + \sigma B_t\right]$$

Earlier we defined GBM as $S_t = S_0 \exp\left(\mu t + \sigma B_t\right)$, thus:

- S_t Geometric Brownian Motion with drift parameter $\mu = \alpha - \frac{1}{2}\sigma^2$ and **volatility** σ.

A Continuous -Time LogNormal Model for Security Prices

Another name for GBM: For $T > t$:
$\log{(S_T)} - \log{(S_t)} \sim N(\mu(T-t), \sigma^2(T-t))$

- μ and σ specific to the investment/security

Stylised facts

Let's collect these stylised facts:

- No linear autocorrelation: $\text{corr}(R_t, R_{t+1}) \approx 0$; (confirms weak form of EMH)
- Volatility clustering: $\text{corr}(R_t^2, R_{t+1}^2) > 0$. We can observe periods of large volatility and of small volatility;
- Heavy tails: High losses and gains much more likely than for normally distributed random variables.

Stochastic Volatility Models

Observation: Time-dependent volatility;

not available in Lognormal model

Approach $X_t \sim \mathcal{N}(\mu, \sigma_t^2)$ (no longer iid),

with stochastic process σ_t

$$X_t = \mu + \sigma_t Z_t, \quad Z_t \sim \mathcal{N}(0,1)$$

With σ_t and Z_t independent

Stationary AR(1) Model

AR(1): autoregressive with dependency on one past value

$$(\sigma_t)_{t\in\mathbb{Z}}: \sigma_t = \alpha + \beta\,\sigma_{t-1} + \nu\epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0,1) \text{ iid}$$

is weakly stationary for $|\beta| < 1$.

- Expectation value: $\mathbb{E}(\sigma_t) = \frac{1}{1-\beta}\alpha$
- Variance: $\mathrm{Var}(\sigma_t) = \frac{1}{1-\beta^2}\nu^2$
- Autocorrelation: $\mathrm{corr}(\sigma_t, \sigma_{t-1}) = \beta$

See next slides

we have $\sigma_t = \alpha + \beta\,\sigma_{t-1} + v\epsilon_t$

$$\Longrightarrow \mathbb{E}(\sigma_t) = \alpha + \beta\mathbb{E}(\sigma_{t-1}) + \nu\mathbb{E}(\varepsilon_t)$$

$= \mu$ $\quad$ $= \mu$ $\quad$ $= 0$

$$\Longrightarrow \mu = \frac{\alpha}{1-\beta} \qquad \text{(and } \alpha = \mu(1-\beta)\text{)}$$

and $\mathrm{Var}(\sigma_t) = \mathrm{Var}(\alpha + \beta\sigma_{t-1} + \nu\varepsilon_t) = \beta^2\,\mathrm{Var}(\sigma_{t-1}) + v^2\,\mathrm{Var}(\varepsilon_t)$

$= \sigma^2$ $\quad$ $= \sigma^2$ $\quad$ $= 1$

$$\Longrightarrow \sigma^2 = \frac{v^2}{1-\beta^2}$$

$\sigma_t = \alpha + \beta \sigma_{t-1} + \nu \epsilon_t$ **How to compute the autocorrelation?**

$$\mathrm{Cov}(\sigma_t, \sigma_{t-1}) = \mathbb{E}[\sigma_t \sigma_{t-1}] - \mathbb{E}[\sigma_t]\mathbb{E}[\sigma_{t-1}]$$

insert eq for σ_t

$$= \mathbb{E}[(\alpha + \beta \sigma_{t-1} + \nu \epsilon_t)\sigma_{t-1}] - \mathbb{E}[\sigma_{t-1}]^2$$

indep. of $\sigma_{t-1}, \varepsilon_t$

$$= \alpha \mathbb{E}[\sigma_{t-1}] + \beta \mathbb{E}[\sigma_{t-1}^2] + \nu \mathbb{E}[\epsilon_t]\mathbb{E}[\sigma_{t-1}] - \mathbb{E}[\sigma_{t-1}]^2$$

$\alpha = \mathbb{E}[\sigma_{t-1}](1-\beta)$,
$\mathbb{E}[\varepsilon_t] = 0$

$$= (1 - \beta - 1))\mathbb{E}[\sigma_{t-1}]^2 + \beta \mathbb{E}[\sigma_{t-1}^2]$$

$$= \beta \left(\mathbb{E}[\sigma_{t-1}^2] - \mathbb{E}[\sigma_{t-1}]^2 \right) = \beta\, \mathrm{Var}(\sigma_{t-1}) = \beta \sigma^2$$

$$\Longrightarrow \mathrm{Corr}(\sigma_t, \sigma_{t-1}) = \frac{\mathrm{Cov}(\sigma_t, \sigma_{t-1})}{\sqrt{\mathrm{Var}(\sigma_t)\mathrm{Var}(\sigma_{t-1})}} = \frac{\beta \sigma^2}{\sigma^2} = \beta$$

Behavioural Finance

Looks at the psychology that underlies and drives financial decision making behaviour.
Helps investors understand how human biases impact on financial decisions and market prices, returns and allocation of resources.

Prospect Theory

- Assumes that people are risk averse when considering gains and risk seeking when considering losses.

Framing and Question-wording

- The wording of a question in terms of gains and losses can have a big impact on the decision made.
- Changing just a word or two can have a profound effect on the answer.

Anchoring and Adjustment
Term used in psychology to describe the common human tendency to rely too heavily, or "anchor" on one piece of information when making decisions.

- Usually, once an anchor is set, there is a bias towards this value.
- The effect of anchoring and adjustment grows with the size of the difference between the anchor value and the pre-anchor estimate.
- Being exposed to high anchors leads to increased mean estimates

Myopic Loss Aversion

- Investors are less risk-averse when faced with a multi-period series of gambles.
  - When the performance of a risky asset is frequently assessed, the probability of detecting a loss is high.

Estimating Probabilities

- Dislike of negative events: people underestimate the probability that negative events may occur.
- Representativeness: people consider those events that they can easily imagine, to be more probable.
- Availability: people are influenced by the ease with which something can be brought to mind.

Overconfidence

- People tend to over-estimate their own knowledge, abilities and skills.
- Discrepancy between accuracy and overconfidence increases as the respondent becomes more knowledgeable.
- Accuracy increases by a small amount, confidence increases to a much larger degree!

Hindsight bias

- Events that have happened will be thought of as having been predictable prior to the event.
- Events that do not happen will be thought of as having been unlikely prior to the event.

Confirmation bias

- People tend to look for evidence that confirms their point of view.
- They tend to dismiss evidence that does not justify their point of view.

Mental Accounting

- People show a tendency to separate related events and find it difficult to aggregate events.

Effect of Options

- Primary effect: People tend to choose the first option.
- Recency effect: People might prefer the final option presented.
- Other research indicates that people might choose an intermediate option!
- Greater range of options discourages decision-making.

Other factors

- Status quo bias: People prefer to leave things unchanged.
- Regret aversion: Retaining existing arrangements to minimise the possibility of regret.
- Ambiguity aversion: People are willing to pay a premium for rules.