MTH5114 **Linear Programming and Games, Spring 2024**

Week 11 Seminar Questions **Viresh Patel**

Practice Exam Question:

Consider the following 2-player zero-sum game. Each player separately chooses a number from the set $\{1, 2, 3\}$. Both players then reveal their numbers. If the numbers match, the row player must pay £3 to the column player, otherwise, the player with the lower number must pay £1 to the player with the higher number.

(a) Give the payoff matrix for this game from the perspective of the row player. Also give the security level for each of the player's strategies.

(b) Does this game possess a pure Nash equilibrium? If so, give all pure Nash equilibria for the game. If not, say why.

(c) Formulate a linear program that finds the row player's best mixed strategy in this game (you do not need to solve this program). [You will be able to do this part of the question at the end of week 11.]

Discussion Questions:

1. Consider the 2-player, zero-sum game "Rock, Paper, Scissors". Each player chooses one of 3 strategies: *rock*, *paper*, or *scissors*. Then, both players reveal their choices. The outcome is determined as follows. If both players choose the same strategy, neither player wins or loses anything. Otherwise:

   - "paper covers rock": if one player chooses paper and the other chooses rock, the player who chose paper wins and is paid 1 by the other player.
   - "scissors cut paper": if one player chooses scissors and the other chooses paper, the player who chose scissors wins and is paid 1 by the other player.
   - "rock breaks scissors": if one player chooses rock and the other player chooses scissors, the player who chose rock wins and is paid 1 by the other player.

   We can write the payoff matrix for this game as follows:

	rock	paper	scissors
rock	0	-1	1
paper	1	0	-1
scissors	-1	1	0

   (a) Show that this game does not have a pure Nash equilibrium.

   (b) Show that the pair of mixed strategies $\mathbf{x}^\mathsf{T} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ and $\mathbf{y}^\mathsf{T} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ together are a Nash equilibrium.

2. Suppose now we alter the game so that whenever Colin chooses "paper" the loser pays the winner 3 instead of 1:

	rock	paper	scissors
rock	0	-3	1
paper	1	0	-1
scissors	-1	3	0

   (a) Show that $\mathbf{x}^{\mathsf{T}} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ and $\mathbf{y}^{\mathsf{T}} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ together are *not* a Nash equilibrium for this modified game.

   (b) Formulate a linear program that can be used to calculate a mixed strategy $\mathbf{x} \in \Delta(R)$ that maximises Rosemary's security level for this modified game.

   (c) Show that $\mathbf{x}^{\mathsf{T}} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ and $\mathbf{y}^{\mathsf{T}} = (\frac{3}{7}, \frac{1}{7}, \frac{3}{7})$ together are a Nash equilibrium for this game.

3. You will be able to do this question after Thursday's lecture in Week 12. Suppose that we further alter the game from question 2 as follows: now whenever both players select the same strategy, *both* receive a payoff of 2. Note that this is no longer a zero-sum game.

   (a) Give the payoff matrix for this game. As usual, you should list Rosemary's payoffs first and Colin's payoffs second in each cell.

   (b) Underline the best responses for each player to each of the other players' strategies in your payoff matrix. Then, find and give all Pure Nash equilibria for the modified game.

4. Consider a 2-player zero-sum game with the following payoff matrix:

	c_1	c_2	c_3
r_1	1	2	-2
r_2	-13	4	12
r_3	1	-7	9

   Give a linear program for computing Rosemary's best strategy (that is, the one that gives her the best security level). Also give a linear program for computing Colin's best strategy.