Queen Mary University of London

Main Examination period 2020 – May – Semester B

MTH4115 / MTH4215: Vectors & Matrices

Duration: 2 hours

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You should attempt ALL questions. Marks available are shown next to the questions.

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Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Examiners: O. Jenkinson, R. Johnson

Question 1 [20 marks]. Let A, B, C be points in 3-space with respective position vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix}$. Determine:

(a) The length of the vector $\mathbf{a}+\mathbf{b}+\mathbf{c}$; **[3]**

(b) A unit vector in the direction of $\mathbf{a}$; **[3]**

(c) $\mathbf{a} \cdot \mathbf{b}$; **[3]**

(d) $\mathbf{a} \times \mathbf{b}$; **[3]**

(e) A vector equation for the line through A and B; **[4]**

(f) A Cartesian equation for the plane containing A, B and C. **[4]**

Question 2 [20 marks].

(a) What is the definition of a **bound vector**? **[2]**

(b) What is the definition of a **free vector**? **[2]**

(c) What does it mean to say that a bound vector **represents** a free vector? **[2]**

(d) What is the definition of a **parallelogram**? **[2]**

(e) State the **parallelogram axiom**. **[2]**

(f) Given free vectors $\mathbf{u}$ and $\mathbf{v}$, explain in terms of a parallelogram how their **sum** $\mathbf{u}+\mathbf{v}$ is defined. **[3]**

(g) Let O be a fixed origin in 3-space, let P and Q be any points in this space, and let R be the point such that the figure $OPQR$ is a parallelogram. Let $\mathbf{p}$ and $\mathbf{q}$ denote the position vectors for P and Q, respectively. Find an expression for the free vector represented by $\overrightarrow{RP}$ in terms of $\mathbf{p}$ and $\mathbf{q}$. **[4]**

(h) With the points as in (g) above, let S be the point such that the figure $ORPS$ is a parallelogram. Find an expression for the free vector represented by $\overrightarrow{SQ}$ in terms of $\mathbf{p}$ and $\mathbf{q}$. **[3]**

Question 3 [20 marks]. Let Π be the plane with equation $2x+y+z=1$, let l be the line with equations $x=y=z$, and let Q be the point with position vector $\mathbf{q}=\begin{pmatrix}0\\1\\2\end{pmatrix}$.

(a) Determine the distance between the point Q and the plane Π. **[4]**

(b) Determine the coordinates of the point on Π that is closest to Q. **[4]**

(c) Determine the distance between the point Q and the line l. **[4]**

(d) Determine the point of intersection of the line l and the plane Π. **[4]**

(e) If l' is the line in the direction orthogonal to Π and passing through Q, then determine the distance between l and l'. **[4]**

Question 4 [20 marks].

(a) What are the three types of **elementary row operation** that can be performed on a matrix? **[3]**

(b) Describe in detail the **Gaussian elimination algorithm** for putting a matrix in row echelon form. **[4]**

(c) Consider the linear system

$$\begin{array}{ccccccccc} 2x_1 & + & 3x_2 & + & x_3 & + & 4x_4 & = & 1 \\ 2x_1 & + & 3x_2 & + & 4x_3 & + & x_4 & = & 0 \\ 2x_1 & + & 3x_2 & - & 2x_3 & + & 7x_4 & = & 2 \\ 2x_1 & + & 3x_2 & + & 4x_3 & - & 2x_4 & = & 1\,. \end{array}$$

(i) Write down the augmented matrix of the system. **[3]**

(ii) Bring the augmented matrix to row echelon form. Indicate which elementary row operation you use at each step. **[6]**

(iii) Identify the leading and free variables of the reduced system, and write down the solution set of the system. **[4]**

Question 5 [20 marks]. Let

$$A = \begin{pmatrix} 0 & -1 & 3 \\ 1 & 0 & -3 \\ 1 & 2 & 0 \end{pmatrix}, \ B = \begin{pmatrix} 3 & -2 \\ -3 & 1 \\ 2 & -1 \end{pmatrix} \text{ and } C = \begin{pmatrix} -2 & -2 & 3 \\ 2 & 1 & 0 \\ -1 & 1 & -2 \end{pmatrix}.$$

(a) For each of the expressions B^2, BA, CB, CA, $A-4B$, and $-A+5C$, state whether or not it exists, and if it exists then evaluate it. **[12]**

(b) Is the matrix $\begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix}$ invertible? Justify your answer. **[2]**

(c) Find a 2×2 matrix D such that $D \neq 0_{2\times 2}$ but $D^2 = 0_{2\times 2}$. **[3]**

(d) Find 2×2 matrices E and F such that $EF = 0_{2\times 2}$ but $FE \neq 0_{2\times 2}$. **[3]**

End of Paper.