1. A shepherd has a utility of wealth $u(w) = w^{\frac{2}{3}}$ for $w > 0$ and $u(0) = 0$.
   a) Show that the shepherd is:
      i) non-satiated and,
      ii) risk averse
   b) Calculate the coefficients of risk aversion and explain what they convey.

Answer:

a) Non satiation is satisfied as: $U'(w) = \frac{2}{3}w^{-\frac{1}{3}} > 0$ for $w > 0$

Risk aversion is satisfied as: $U''(w) = -\frac{2}{9}w^{-\frac{4}{3}} < 0$ for $w > 0$

b) $A(w) = -\frac{-\frac{2}{9}w^{-\frac{4}{3}}}{\frac{2}{3}w^{-\frac{1}{3}}} = \frac{1}{3w}$

$A'(w) = -\frac{1}{3}w^{-2} < 0$ for $w > 0$. Hence the investor exhibits decreasing absolute risk aversion. This means that as his wealth increases the absolute amount of wealth invested in risky assets increases

$$R(w) = -\frac{-\frac{2}{9}w^{-\frac{4}{3}}}{\frac{2}{3}w^{-\frac{1}{3}}}w = \frac{1}{3}$$

This is a constant relative risk aversion utility function as $R'(w) = 0$, which means that as this investor's wealth increases the relative amount of wealth in risky assets stays constant.

The shepherd begins every year with a herd of sheep with value £100,000, but over the year, she loses some sheep due to wolf attacks. If the winters are hard, she loses 0.25% of her herd, and if the winters are mild, she loses 0.05% of her herd. Every year there are 50-50 chances of either hard or mild winter. The shepherd is considering buying a sheepdog that will protect the herd in case of wolf attack. A fellow shepherd tells her that there are three type of breeds that are the best for this task: Border Collie, Groenendael and Berger de Brie. These breeds are guaranteed to protect fully against any wolf attack.

The shepherd has done some research about the three breeds and she summarised her findings in the Table below:

	Border Collie	Groenendael	Berger de Brie
Current price per dog	£1,200	£1,300	£1,400
Average working life before retirement	10 years	13 years	7 years

If the shepherd buys the dog there is an agreement in place with the sellers to spread the cost over the average working life of the dog. Note that this is a zero-interest agreement between buyers and sellers.

c) Calculate the average expected annual return for the three dogs.

Answer:

By buying a dog the shepherd will not incur the losses from wolf attacks. So this will be used as the annual profit from having a dog.

	Hard winter	Mild winter
Annual profit from owning a dog per year	250	50
Average annual investment per year for a Border Collie	120	120
Average annual investment for a Groenendael	100	100
Average annual investment for a Berger de Brie	200	200

	Hard winter with prob 50%	Mild winter with prob 50%
Average year return for Border Collie	$\frac{250}{120} - 1 = 1.0833$	$\frac{50}{120} - 1 = -0.58333$
Average year return for a Groenendael	$\frac{250}{100} - 1 = 1.5$	$\frac{50}{100} - 1 = -0.5$
Average year return for a Berger de Brie	$\frac{250}{200} - 1 = 0.25$	$\frac{50}{200} - 1 = -0.75$

Hence the expected annual returns for each of the dogs are:

	Border Collie	Groenendael	Berger de Brie
Expected annual return	0.25 or 25%	0.5 or 50%	-0.25 or -25%

d) Calculate the standard deviation of the returns for these dogs.

$$Var\ (BC) = 0.5 \times (1.0833 - 0.25)^2 + 0.5 \times (-0.58333 - 0.25)^2 = 0.6944$$
$$Var\ (Gr) = 0.5 \times (1.5 - 0.5)^2 + 0.5 \times (-0.5 - 0.5)^2 = 1$$
$$Var\ (BB) = 0.5 \times (0.25 + 0.25)^2 + 0.5 \times (-0.75 + 0.25)^2 = 0.5$$

The standard deviation is the square root of above.

The summary statistics of returns are:

	Mean	Variance	St. Dev.
Border Collie	0.25	0.694444	0.83333333
Groenendael	0.5	1	1
Berger de Brie	-0.25	0.25	0.5

The shepherd decides which dog to buy based on a risk measure r is, defined as the ratio between expected annual return and standard deviation of returns.

e) Provide a rational behind the investor's choice of the performance measure r.

Answer:

The investor's choice of measure r, is the expected return per unit of risk (in this case standard deviation). This can be interpreted as a risk-adjusted performance measure. Note that this is called the Sharpe ratio (we did not define it in this module but you may know it from other classes). No points will be awarded for knowing the name.

The investor wants to choose a portfolio which will maximise this measure. Students need to mention that r increases when there is:

i. an increase in expected return,
ii. or decrease in risk,

This is consistent with MVPT

f) Find which dog the shepherd buys.

	Mean	Variance	St. Dev.	Sharp Ratio
Border Collie	0.25	0.694444	0.83333333	0.3
Groenendael	0.5	1	1	0.5
Berger de Brie	-0.25	0.25	0.5	-0.5

Based on the (r) the Shepherd buys a Groenendael.

2. The shepherd from question 1, decides to insure her sheepdog against theft and physical damage. One of the most common medical problems of sheepdogs is retinal atrophy and the shephard is looking for an insurance to cover this problem as well. Her estimate of the amount of loss and the associated probabilities are as given in the table below:

Type of loss and value	Probability
Theft: £1,000	0.2
Retinal atrophy: £800	0.2
Other medical problems: £200	0.3

An insurance company with a capital of £100,000 appraises the potential of losses and their associated probabilities in the same way as the horse owner. The utility function of the insurance company is $u(c) = 10{,}000 + 0.5\,c$, where $c > 0$ is the insurer's capital.

a) Calculate the minimum premium the insurer is willing to offer for a policy that insures the above losses.

b) Calculate the maximum price that the dog owner is prepared to pay in order to insure her dog.

c) Determine whether the insurance company and the dog owner can agree on policy that is advantageous to both parties.

Answer:

b) The choice faced by the insurer is whether to offer insurance for a premium (take gamble) or not offer insurance at all (not take gamble). Minimum premium is Q which is received in all possible states (not random). The loss is random: $X = \{1{,}000; 800; 200; 0\}$ with respective probabilities: $\{0.2;\ 0.2; 0.3; 0.3\}$
The premium is determined by:

$$EU(c_0 + Q + X) = U(c_0)$$

$$EU(c_0 + Q + X) = 0.2[10{,}000 + 0.5\,(100{,}000 + Q - 1{,}000)]$$

$$+0.2[10{,}000 + 0.5\,(100{,}000 + Q - 800)]$$

$$+0.3[10{,}000 + 0.5\,(100{,}000 + Q - 200)]$$

$$+0.3[10{,}000 + 0.5\,(100{,}000 + Q)]$$

$$EU(c_0 + Q + X) = [10{,}000 + 0.5\,(100{,}000 + Q)]$$

$$-0.2 \times 0.5 \times 1{,}800 - 0.3 \times 0.5 \times 200$$

$$= 59{,}790 + 0.5Q$$

$$U(c_0) = 10{,}000 + 0.5 \times 100{,}000 = 60{,}000$$

The indifference condition is :

$$EU(c_0 + Q + X) = U(c_0)$$

or:

$$59{,}790 + 0.5Q = 60{,}000$$

Hence, $Q = 420$

c) Maximum price the dog owner is willing pay is c_x such that $\mathrm{U}(\mathrm{w}_0 - \mathrm{c}_\mathrm{x}) = \mathrm{E}[\mathrm{U}(\mathrm{w}_0 + \mathrm{X})]$ where w_0 is the initial wealth of 100,000 and X is the random loss.

Thus, in order to find c_x:

$$U(w_0) = (100{,}000 - c_x)^{2/3}$$

$$EU(w_0 + X) = 0.2 \times (100{,}000 - 1{,}000)^{2/3} + 0.2 \times (100{,}000 - 800)^{2/3} + 0.3 \times (100{,}000 - 200)^{2/3} + 0.3 \times (100{,}000)^{2/3} = 2{,}148.394$$

$$(100{,}000 - c_x)^{\frac{2}{3}} = 2{,}148.394$$

$$100{,}000 - c_x = 99{,}759.73$$
$$c_x = 420.274$$

If the premium offered by the insurer is between 420 and 420.274 so the parties will agree and the deal will be advantageous to both parties

3. The forecast for the economic outlook is that, the economy will be either in recession with probability 0.25, or experience normal growth with probability 0.75. There are two stocks available to trade in this economy (stock SeaLab and stock BinTech). The stocks' annual returns in each of the states of the world are according to the following table:

Stocks	Recession	Normal Growth
SeaLab	8%	12%
BinTech	16%	10%

a) Show whether you can rank the stocks by the criterion of first order stochastic dominance. If yes, which stock first order stochastically dominates the other?

b) Show whether you can rank the stocks by the criterion of second order stochastic dominance. If yes, which stock second order stochastically dominates the other?

c) Calculate the expected annual return of each stock.

d) Calculate the annual variance and standard deviation of each stock.

e) Calculate the covariance and correlation coefficient between the two stocks.

f) An investor plans to set up a risk-free portfolio with these two stocks. Determine what weights he needs to put on each of the stocks such that he achieves this goal.

g) Assuming that the CAPM holds calculate the composition of the market portfolio with expected return 10% per annum.

h) Calculate the beta of each security, under the assumption that the risk-free rate of interest is 1% per annum and interpret your results.

Answers

a)

Stocks	Recession	Normal Growth
SeaLab	8% with prob 0.25	12% with prob 0.75
BinTech	16% with prob 0.25	10% with prob 0.75

Returns	SeaLab pdf	SeaLab cdf	BinTech pdf	BinTech cdf
8%	0.25	0.25 >	0	0
10%	0	0.25 <	0.75	0.75
12%	0.75	1 >	0	0.75
16%	0	1 =	0.25	1

In order to have one asset first order dominating another we need at every possible return, the cdf of returns of one asset to be less or equal to cdf of returns of the other asset with at least one strict inequality. In this case we see that this is not the case and we cannot first order dominance rank these assets

b)

Returns	SeaLab cdf	Sea sum cdf	BinTech cdf	Bin sum cdf
8%	0.25	0.25 >	0	0
10%	0.25	0.5<	0.75	0.75
12%	1	1.5 =	0.75	1.5
16%	1	2.5 =	1	2.5

Hence we cannot rank these stocks also on second order stochastic dominance.

c)

$$E(R_{Sea}) = 0.08 \times 0.25 + 0.12 \times 0.75 = 0.11 = 11\%$$
$$E(R_{Bin}) = 0.16 \times 0.25 + 0.10 \times 0.75 = 0.115 = 11.5\%$$

d)

$$Var(R_{Sea}) = (0.08 - 0.11)^2 \times 0.25 + (0.12 - 0.11)^2 \times 0.75 = 0.0003$$
$$Stdv(R_{Sea}) = 0.026$$

$$Var(R_{Bin}) = (0.16 - 0.115)^2 \times 0.25 + (0.10 - 0.115)^2 \times 0.75 = 0.000675$$
$$Stdv(R_{OBN}) = 0.0732$$

e)

$$Cov(R_{MJ}, R_{OBN}) = (0.08 - 0.11) \times (0.16 - 0.115) \times 0.25 + (0.12 - 0.11) \times (0.10 - 0.115) \times 0.75 = -0.00045$$
$$Corr(R_{Sea}, R_{Bin}) = -\frac{0.00045}{0.026 \times 0.0732} = -1$$

g) these assets are perfectly negatively correlated hence it is possible to get a risk free asset. The minimum global variance portfolio in this case will have variance zero with weights:

$$w_{Sea} = \frac{\sigma_{Bin}^2 - \rho\sigma_{Sea}\sigma_{Bin}}{\sigma_{Sea}^2 + \sigma_{Bin}^2 - 2\rho\sigma_{Bin}\sigma_{Sea}} = 0.6$$

$$w_{Bin} = 0.4$$

h)

There are only two assts in this economy, so the supply side represented by the market portfolio is formed by these two assets. Let w_{Sea} be the weight on stock SeaLab and $w_{Bin} = 1 - w_{Sea}$ the weight on stock Bintech.
Hence,

$$E(R_M) = w_S E(R_{Sea}) + w_{Bin} E(R_{Bin})$$
$$0.1 = 0.11 w_{Sea} + 0.115(1 - w_{Sea})$$
$$0.005 w_{Sea} = 0.015$$
$$w_{Sea} = 0.33$$
$$w_{Bin} = 0.67$$

i) CAPM: $E(R_i) = r + \beta_i(E(R_M) - r)$

$$\beta_{Sea} = \frac{0.11 - 0.01}{0.1 - 0.01} = 1.11$$

$$\beta_{Bin} = \frac{0.115 - 0.01}{0.1 - 0.01} = 1.17$$

Both betas are higher than 1 which mean that these are aggressive stocks or high beta stocks: 1% increase in the market risk premium increases the stock premium by more than 1% .

5)

Which forms of market efficiency are satisfied and/or violated in the following situations:

(a) The host of TV show E-Investments gives stock recommendations every day and insists following these recommendations will beat the market.

You need to explain the three levels of market efficiency first:

A market is weak-form efficient if the strategies based on historical price information do not generate excess profits.
A market is semi-strong-form efficient if prices reflect (or incorporate) all publicly available information, such as, published accounting statements for the firm as well as historical price information.
A market is strong-form efficient if prices reflect all information public or private.

Answer: If his/her strategies are indeed profitable than the market is not efficient since everyone has access to them. We don't know based on what the TV host makes her recommendation (past stock data, private information, etc) so we cannot comment on the form of efficiency violated.

(b) A pharmaceutical company announces that it has discovered a vaccine for violet fever. Its share price rises immediately by 12%.

Answer: It appears that the market responds swiftly to an announcement, suggesting that it is consistent with the semi-strong form efficiency. The market is unlikely to be strong form efficient as the privately held information is not already in the price

(c) If a firm's stock price falls by more than 1% on any given day, the return is typically positive the following day.

Answer: This is negative autocorrelation which implies that past returns are not fully incorporated in the stock price. This is a violation of weak form efficiency. Thus also not strong form efficient.

5) The trustees of a charity that supports research into violet fever, have approached a financial consultant to find out about behavioural finance and its relevance to the trustees' work.

a. Briefly discuss what is meant by behavioural finance

Answer: The field of behavioural finance looks at how a variety of mental biases and decision- making errors affect financial decisions. It relates to the psychology that

underlies and drives financial decision-making behaviour. (Credit can also given for: it helps investors understand how human biases impact on financial decisions and market prices, returns and allocation of resources.)

b. State, giving reasons, two examples of behavioural finance themes that are relevant to the trustees of the charity.

Answer: The following solution gives more detail than what is required to score full marks.

Anchoring and Adjustment – The trustees may rely too heavily, or "anchor" on one piece of information when making decisions relating to the charity. Usually, once an anchor is set, there is a bias towards this value, irrespective of the accuracy of the anchor. For example, the trustees may exhibit this theme when negotiating fees/charges by their investment managers – the managers could provide an anchor (fee level) much higher than what the trustees were expecting and, unless they shop around, the trustees may end up agreeing to these higher levels of fees.

Myopic loss aversion states that when the performance of a risky asset is frequently assessed, the probability of detecting a loss is high. This puts off risk-averse investors. However, investors are less risk-averse when faced with a multi-period series of gambles. For example, trustees are more likely to choose more volatile assets like equities when they are presented with a longer period of equity returns (say annual returns over the last 25 years) instead of daily or weekly returns. The more visible volatility in the daily or weekly returns may discourage them from choosing equities and they may then prefer to invest in less volatile assets, like bonds.

Overconfidence – The trustees may tend to over-estimate their own knowledge, abilities and skills and make decisions based on their assumed knowledge instead of relying on expert professional help in matters such as investment of the charity's funds.

Effect of Options – When a number of different options are presented, for example, in relation to investment opportunities for the charity fund, the number, order and degree of difference between the options will all affect which option will be chosen by the trustees. This may lead to sub-optimal choice selection.

Any other reasonable example/theme will also be given credit if correctly explained. One mark for each example. Two marks for correctly naming and describing each theme.

c. Suggest reasons for the trustees' interest in learning about behavioural finance.

Answer: Understanding the impact of behavioural finance on prices in capital markets can help the trustees avoid errors commonly made by other investors.

The trustees may also be able to take advantage of errors made by other investors to invest more profitably.

The trustees may have been forced by regulators to understand and be aware of how certain behavioural themes impact on their performance as trustees.