Queen Mary University of London

MTH5114 **Linear Programming and Games, Spring 2024**

Week 10 Seminar Questions **Viresh Patel**

Practice Exam Question:

Consider the following 2-player zero-sum game. Each player separately chooses a number from the set $\{1, 2, 3\}$. Both players then reveal their numbers. If the numbers match, the row player must pay £3 to the column player, otherwise, the player with the lower number must pay £1 to the player with the higher number.

(a) Give the payoff matrix for this game from the perspective of the row player. Also give the security level for each of the player's strategies.

(b) Does this game possess a pure Nash equilibrium? If so, give all pure Nash equilibria for the game. If not, say why.

(c) Formulate a linear program that finds the row player's best mixed strategy in this game (you do not need to solve this program). [You will be able to do this part of the question at the end of week 11.]

Discussion Questions:

1. (a) Give an example of a 2-player zero-sum game in which the row player (Rosemary) has 2 strategies, the column player (Colin) has 3 strategies and every pair of strategies is a Nash equilibrium (i.e. there are six Nash equilibria.) Exhibit your example by giving the payoff matrix (as usual from the row player's perspective).

   (b) Following on from the previous question, for each $n = 0, 1, \dots, 6$ give an example of a 2-player zero-sum game with the following properties or explain why one does not exist. The row player (Rosemary) has 2 strategies, the column player (Colin) has 3 strategies and there are exactly n Nash equilibria. Note that the previous question is the case $n = 6$ of this question.

2. Consider an arbitrary 2-player zero-sum game where Rosemary's set of strategies is $\{r_1, r_2\}$ and Colin's set of strategies is $\{c_1, c_2\}$, and the payoff to Rosemary when Rosemary plays r_i and Colin plays c_j is the number $a_{ij} \in \mathbb{R}$.

   (a) Write down the payoff matrix for this game.

   (b) Suppose that Rosemary uses a mixed strategy $(p, 1-p)$ and Colin uses a mixed strategy $(q, 1-q)$, where $p, q \in [0, 1]$. What is the expected payoff to Rosemary and what is the payoff to Colin.

(c) By writing Colin's mixed strategy as

$$\begin{pmatrix} q \\ 1-q \end{pmatrix} = q \begin{pmatrix} 1 \\ 0 \end{pmatrix} + (1-q) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

show that Colin can improve[1] his expected payoff by using one of his pure strategies instead of his mixed strategy $\mathbf{q} = (q, (1-q))$ assuming that Rosemary sticks with her mixed strategy $\mathbf{p} = [(p, (1-p)$.

[1]Here, by improve, we mean "at least as good as" so might not be a strict improvement