Queen Mary University of London

MTH5114 Linear Programming and Games, Spring 2024

Week 9 Seminar Questions **Viresh Patel**

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1. Use the principle of complementary slackness to determine whether or not $\mathbf{x}^\mathsf{T} = (0, 4, 0, 2)$ is an optimal solution to the following linear program.

$$\begin{aligned} \text{maximize} \quad & 9x_1 + 3x_2 + 5x_3 + 22x_4 \\ \text{subject to} \quad & 2x_1 - x_2 + 2x_3 + 6x_4 \leq 8, \\ & 5x_1 + 3x_2 + x_3 + 2x_4 \leq 16, \\ & 4x_1 + x_2 - x_3 + 3x_4 \leq 12, \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

2. Use the principle of complementary slackness to determine whether or not $\mathbf{x}^\mathsf{T} = (0, \frac{5}{6}, \frac{3}{2})$ is an optimal solution to the following linear program.

$$\begin{aligned} \text{maximize} \quad & \tfrac{1}{2}x_1 + x_2 + x_3 \\ \text{subject to} \quad & 2x_1 + 7x_2 + x_3 \leq 8, \\ & x_1 + 3x_2 + 3x_3 \leq 7, \\ & 2x_1 \quad\quad + 4x_3 \leq 6, \\ & x_1 + 3x_2 + \ x_3 \leq 9, \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Practice Exam Question: A power company operates three power generation plants. One is a wind plant, and the other two consume a combination of Fuel 1 and Fuel 2, emitting carbon dioxide in the process. In addition, all three plants require maintenance. The amount of fuel consumed (in Mg), maintenance required (in person-hours), carbon dioxide (CO_2) emitted (in Mg), and power generated (in MWh) per day of operation is as follows:

Plant	Maintenance Required	Fuel 1 Required	Fuel 2 Required	CO_2 Emitted	Power Produced
1	20	0	0	0	20
2	13	10	15	12	32
3	18	30	40	29	40

Each MWh of power can be sold at £121 and there is no limit on the amount that can be sold. Over its next planning period, the company has 230 person-hours for maintenance, 75 Mg of Fuel 1, and 90 Mg of Fuel 2 available.

(a) Due to environmental regulations, they cannot emit more than 200Mg of CO_2 in this period. The company wants to know how to operate its plants to generate as much revenue as possible. (You may assume that there is no limit on the number of days a plant can operate in this period). Give a linear program that models this problem. You do not need to solve this program.

(b) Suppose now that the company can emit more than 200Mg of CO_2, but now loses £55 of revenue for each Mg emitted after the first 200Mg because it must purchase "CO_2 credits". The other resource constraints remain as stated. The company now wants to know how to operate its plants to generate as much revenue as possible (where revenue should now take account of this extra cost). (Again, you may assume that there is no limit on the number of days that a plant can operate in this period.) Give a linear program that models this problem. You do not need to solve this program.