Mathematical Tools for Asset Management

The Efficient Markets Hypothesis

Theory and Empirical Evidence

Dr. Melania Nica

Plan

- Efficient Market Theory
- Different Forms of Market Efficiency
  - Weak Form Efficiency: Implications and Tests
  - Semi-strong Form Efficiency: Implications and Tests
  - Strong Form Efficiency: Implications and Tests
- Moving from Discrete Time to Continuous Time
  - EMH and the Random Walk Model of Stock Prices
  - Generating a Continuous Random Walk
  - Brownian Motions and Stock Prices

Efficient Market Theory

Jensen (1978):
A market is efficient with respect to a given information set Ω if no agent can make economic profit through the use of a trading rule based on Ω.

- economic profit: the level of return after costs are adjusted appropriately for risk

Efficient Market Hypothesis (EMH)**:** stock prices already reflect all available information, hence:

- **changes in prices** should be unpredictable (random)

Efficient Market Theory

EMH: if prices are determined rationally, then only new information will cause them to change

- this new information is should be unpredictable in an efficient (i.e. well-functioning) market

What if part of the "new information" is predictable?

Efficient Market Theory

Why should we expect stock prices to reflect "all available information"?

- **equilibrium argument:** competition to capture expected return will drive away any inefficiency in the market

But, this argument relies on two critical assumptions

- **perfect competition**
- **instantaneous price adjustment**

Efficient Market Theory

Investors have an incentive to look for new information only if it generates higher investment returns.

- in market equilibrium, efficient information-gathering activity should be fruitful.
- the magnitude of this “fruit” depends on the degree of disequilibrium existing in the market.

Efficient Market Theory

The degree of efficiency varies across financial markets

- emerging markets are less intensively analyzed than the U.S. markets
- small stocks that receive relatively little coverage by Wall street analyst may be less efficiently priced than large ones.

Efficient Market Theory

Fama (1991):
EMH has three different versions based on definition of "all available information:"

- **Weak-form hypothesis**: stock prices already reflect all information that can be derived by examining *market trading data* such as historical prices, volumes, etc.
- **Semi-strong form hypothesis**: stock prices. incorporate *all publicly available information* regarding the firm's prospects (*market trading data* + future projects, earning forecasts)
- **Strong form hypothesis:** stock prices reflect *all information relevant to the firm* (including private information of company insiders + *all publicly available information*)
- If market is not weakly efficient, then it is not semi-strong efficient then it is not strong efficient.

Efficient Market Theory

- Eugene Fama, Robert Shiller and Lars Peter Hansen got the Nobel prize in economics in 2013 for research on how financial markets work and assets are priced

Efficient Market Theory

Risk- adjusted or abnormal or excess returns: between the actual returns on stock i and expected returns

$$r_t^X = r_t - E(r_t)$$

r_t^X is excess return and r_t is the actual stock return at time t
$E(r_t)$ obtained through:

- CAPM, the APT or a less theory-motivated choice
- market model, which estimates the expected return of stock i through a regression of stock i's actual returns on those of the market
- naïve method: assume expected returns are constant

EMH: **Can we make excess returns based on a certain information set?**
If Markets are efficient: No
How we test this?

Efficient Market Theory

Keep in mind: we *do not know the true model* that generates expected returns in the economy

- abnormal returns may be incorrectly measured
- these (inaccurate) abnormal returns are then used in tests of market efficiency

Efficient Market Theory

The null hypothesis of any test of efficiency is comprised of two components:

- informational efficiency
- the accuracy of one's model for expected returns

A *rejection of this null hypothesis cannot be immediately taken as evidence that markets are not efficient!!*

Weak Form Efficiency: Implications and Tests

Current and past asset returns should have no predictive power for future returns on that asset

One cannot form a trading rule based on current and historical returns

Statistically:

$$E(r_{t+1}|r_t, r_{t-1}, r_{t-2}, r_{t-3}, \ldots) = 0$$

which implies that returns are uncorrelated with their own past values:

$$Cov(r_t, r_{t-s}) = 0, s > 0$$

You can test it!

Weak Form Efficiency: Implications and Tests

The random walk model

$$\begin{aligned} P_t &= P_{t-1} + \varepsilon_t \\ P_t - P_{t-1} &= \varepsilon_t \\ E(\varepsilon_t) &= 0, Cov(\varepsilon_t, \varepsilon_s) = 0, t \neq s \end{aligned}$$

where P_t is log of stock price in t

The change in *log price* from time $t-1$ to t is a mean zero, serially uncorrelated innovation, ε_t

- ε_t new public information arriving at market during period t.

Weak Form Efficiency: Implications and Tests

Past price changes carry no information about current or future price changes.

Stock price return is the first difference of the log stock price

$$r_t = P_t - P_{t-1}$$

and

$$r_t = \varepsilon_t$$

Tests of return autocorrelation can be viewed as tests of the random walk model.

Weak Form Efficiency: Implications and Tests

Weak-form efficiency is also be violated if any information available at time t or before allowed us to forecast returns.

$$r_{t+1} = \alpha + \beta X_t + \varepsilon_t, E(\varepsilon_t) = 0, Var(\varepsilon_t) = \sigma^2$$

X_t is the forecasting variable for returns
ε_t is a regression error term.
Weak-form efficiency would imply that the coefficient β in above equation should be statistically insignificant.

Weak Form Efficiency: Implications and Tests

A calendar effect is defined as a pattern in stock returns related to either the day of the week, the week of the month or the month of the year.

Day-of-the-week effect

- Mondays are historically bad days for stock returns. Wednesday and Fridays are consistently good days for stock returns. Tuesday and Thursdays are mixed bags.

Weekend effect

- Mondays are bad and Fridays are good.
- does anything bad happen over weekends?

Weak Form Efficiency: Implications and Tests

Turn-of-the-calendar effect

- most returns come from the end and beginning trading days; middle of month returns are almost zero.

January Effect:

- stock returns tend to inexplicably high in the month of January. Moreover the return on small-size firms is higher compared to large-small firms

These facts established along time back, yet they are persisting even today.

Weak Form Efficiency: Implications and Tests

$$r_{t+1} = \alpha + \beta X_t + \varepsilon_t, E(\varepsilon_t) = 0, Var(\varepsilon_t) = \sigma^2$$

X_t is dummy variable (or set of dummy variables) that picks out the desired calendar effect

	January return	*January return minus average monthly return in rest of year*	*January return after adjusting for systematic risk*
S&P 500 Companies			
Highly Researched	2.48%	1.63%	-1.44%
Moderately Researched	4.95%	4.19%	1.69%
Neglected	7.62%	6.87%	5.03%
Non-S&P 500 Companies			
Neglected	11.32%	10.72%	7.71%

Weak Form Efficiency: Implications and Tests

January effect

- a trading rule that indicated that one should buy (small) stocks at the end of December and sell them at the end of January would make money

Potential explanations for the January effect include:

- taxation impacts
- year-end effects
- effects from the remuneration packages of fund managers

Rational agents should eliminate these types of effects.

Impact of other variables on stock returns
Certain accounting ratios

Is it possible to beat the market by choosing shares whose price is low relative to fundamentals such as earnings, dividends, the book value of equity, or cash-flows?

- Value portfolios (stocks with lowest price to fundamental) outperforms the glamour portfolios
- The excess returns of value over glamour stocks persistent
- Lakonishok, Shleifer and Vishny (1994)

Impact of other variables on stock returns

Past performance as measured by excess returns in prior years

- Portfolios of previous 'losers' are found to subsequently outperform previous 'winners'
- DeBondt and Thaler (1985)

Impact of other variables on stock returns

- *Technical trading rule applications*
- finance practitioners rely on technical trading rules to generate trading signals
- however, if a trading rule actually did generate profits, then its adoption by the masses would eliminate the gains it had generated in the past
- test of this argument done by Brock, Lakonishok and LeBaron (1992) and Levich and Thomas (1993)

Semi-strong Form Efficiency: Implications and Tests

Semi-strong form efficiency:

- stock prices. incorporate *all publicly available information* regarding the firm's prospects (*market trading data* + future projects, earning forecasts)

Semi-strongefficiency is concerned with the speed at which new information is incorporated into asset prices.
The effect of earnings announcements on stock prices

- As earnings announcements reflect the financial health of a firm, we would expect stock prices to rise upon the announcement of better-than-expected earnings (good news) and fall if earnings are below expectations (bad news.)
- Event studies
- There is evidence both consistent and inconsistent with semi-strong form of efficiency.

Strong Form Efficiency: Implications and Tests

Empirical studies examine whether corporate insiders (e.g. company directors) make gains from trading in their own company's stock.

- results: insider trades can be used to predict subsequent stock price changes, and hence such work concludes that markets are not strong-form efficient.

Other work shows there is information in the forecasts of professional analysts and surveys

- test on mutual fund performance shows that actively managed portfolios underperform other broad-based portfolios with similar risk.

Blake and Timmermann 1992: mutual funds in the UK have underperformed the market by 2% per year over 23 years

EMH and the Random Walk Model of Stock Prices

- EMH: if prices are determined rationally, then only new information will cause them to change
  - the random walk model of stock prices:

$$P_t = P_{t-1} + \varepsilon_t, E(\varepsilon_t) = 0, Cov(\varepsilon_t, \varepsilon_s) = 0, t \neq s$$

-
  - where P_t is log of stock price in t
- ε_t new public information arriving at market during period t.
- Past history is fully reflected in the present price

Moving From Discrete Time to Continuous Time

- **Brownian motion** is a random walk occurring in continuous time
  - *with movements that are continuous rather than discrete*.
- A random walk can be generated by flipping a coin each period and moving one step
  - *with direction determined by whether the coin is heads or tails*.
- To generate Brownian motion, we would flip the coins infinitely fast and take infinitesimally small steps at each point.

The Simple Symmetric Random Walk

Consider the time interval $(0, 1)$.
Sub-divide $(0, 1)$ into n equal sub-intervals.
For each sub-interval $i = 1, 2, \ldots, n$, define a step:

$$Z_i = \begin{cases} +1 & \text{with probability } 0.5 \\ -1 & \text{with probability } 0.5 \end{cases}$$

Then a **Simple Symmetric Random Walk** is defined as:

$$B_n(1) = \sum_{i=1}^{n} Z_i.$$

The Simple Symmetric Random Walk

$$\begin{aligned} E\left(Z_i\right) &= 0.5 - 0.5 = 0 \\ E\left[B_n\left(1\right)\right] &= 0 \end{aligned}$$

$$\begin{aligned} Var\left(Z_i\right) &= \left(1-0\right)^2 \times 0.5 + \left(-1-0\right)^2 \times 0.5 = 1 \\ Var\left[B_n\left(1\right)\right] &= Var\left[\sum_{i=1}^{n} Z_i\right] = nVar\left[Z_i\right] + 0 = n \end{aligned}$$

Modified Simple Symmetric Random Walk

Consider the time interval $(0, 1)$.
Sub-divide $(0, 1)$ into n equal sub-intervals.
For each sub-interval $i = 1, 2, \ldots, n$, define a step:

$$Z_i^* = \begin{cases} +1/\sqrt{n} & \text{with probability } 0.5 \\ -1/\sqrt{n} & \text{with probability } 0.5 \end{cases}$$

Then a **Modified Simple Symmetric Random Walk** is defined as:

$$B_n(1) = \sum_{i=1}^{n} Z_i^*.$$

Modified Simple Symmetric Random Walk

$$
\begin{aligned}
E\left[Z_i^*\right] &= \frac{1}{\sqrt{n}} \times 0.5 - \frac{1}{\sqrt{n}} 0.5 = 0 \\
E\left[B_n(1)\right] &= 0 \\
Var\left[Z_i^*\right] &= \left(\frac{1}{\sqrt{n}} - 0\right)^2 \times 0.5 + \left(-\frac{1}{\sqrt{n}} - 0\right)^2 \times 0.5 = \frac{1}{n} \\
Var\left[B_n(1)\right] &= Var\left[\sum_{i=1}^{n} Z_i^*\right] = n \times \frac{1}{n} + 0 = 1
\end{aligned}
$$

Generalisation to any time-interval $(0, t)$

- $(0, t)$ is the collection $\{(0, 1) \cup (1, 2) \cup \ldots \cup (t - 1, t)\}$, where
- each interval is again sub-divided into n equal sub-intervals.
- Position of the walk at time t, will be determined by nt steps.
- A general modified simple symmetric random walk is defined as:

$$B_n(t) = \sum_{i=1}^{nt} Z_i^*.$$

- Find $E[B_n(t)]$ and $V[B_n(t)]$.

Generalisation

$$B_n(t) = \sum_{i=1}^{nt} Z_i^*$$

$$\begin{aligned}
E\left[Z_i^*\right] &= \frac{1}{\sqrt{n}} \times 0.5 - \frac{1}{\sqrt{n}} 0.5 = 0 \\
E\left[B_n(t)\right] &= 0 \\
Var\left[Z_i^*\right] &= \left(\frac{1}{\sqrt{n}} - 0\right)^2 \times 0.5 + \left(-\frac{1}{\sqrt{n}} - 0\right)^2 \times 0.5 = \frac{1}{n} \\
Var\left[B_n(t)\right] &= Var\left[\sum_{i=1}^{nt} Z_i^*\right] = nt \times \frac{1}{n} + 0 = t
\end{aligned}$$

Observations on Convergence

Observations

- All the marginal distributions tend towards the same underlying normal structure.
- The future movement, $B_n(t) - B_n(s)$, away from a particular time, s, and position, $B_n(s)$, is independent of where that position is;
- and independent of its entire history up to that time $\{B_n(r) : r \leq s\}$.
- By Central Limit Theorem any future displacement, $B_n(t) - B_n(s) \quad \overset{n\to\infty}{\longrightarrow} \quad N(0, t-s)$.

Result

The distribution of $B_n(t)$ converges towards **Standard Brownian Motion**.

Standard Brownian Motion

Standard Brownian Motion, SBM, also called the Wiener Process, is a stochastic process $\{B_t : t \geq 0\}$, with state space $S = \mathbb{R}$ (set of real numbers) and the following defining properties:

- $B_0 = 0$
- **Independent increments:** $B_t - B_s$ is independent of $\{B_r : r \leq s\}$, where $s < t$
- **Stationary increments:** Distribution of $B_t - B_s$ depends only on $(t - s)$, where $s < t$
  - the change in the value of the process over any two non-overlapping periods are statistically independent
- **Gaussian increments:** $B_t - B_s \sim N(0, t - s)$
- **Continuity:** B_t has continuous sample paths

Brownian Motion: What We Know

Definition

Brownian Motion, BM, is a stochastic process W_t, with state space $S = R$ (set of real numbers) and the following defining properties:

1. Independent increments: $W_t - W_s$ is independent of $\{W_r : r \leq s\}$, where $s < t$.
2. Stationary increments: Distribution of $W_t - W_s$ depends only on $(t - s)$, where $s < t$.
3. Gaussian increments: $W_t - W_s \sim N(\mu(t-s), \sigma^2(t-s))$.
4. Continuity: W_t has continuous sample paths.

Relationship between SBM and BM

- SBM can be obtained from BM by setting $\mu = 0$, $\sigma = 1$ and $W_0 = 0$.
- W_t (BM) can be obtained from B_t (SBM) by $W_t = W_0 + \mu t + \sigma B_t$
- μ - drift parameter and σ- volatility
- A **Geometric Brownian Motion** (GBM) is $S_t = \exp\left(W_t\right) = S_0 \exp\left(\mu t + \sigma B_t\right)$

Brownian Motion As A Stock Model