MTH5131 Actuarial Statistics

Coursework 5

This coursework is not to be turned in. You may ask questions about the coursework in tutorial or by email.

Exercise 1. The table below shows annual aggregate claim statistics for 3 risks over 8 years. Annual aggregate claims for risk i, in year j are denoted by X_{ij}

Risk, i	$\bar{X}_i = \frac{1}{8}\sum_{j=1}^{8} X_{ij}$	$s_i^2 = \frac{1}{3}\sum_{j=1}^{8}\left(X_{ij} - \bar{X}_i\right)^2$
1	213.11	411.9
2	91.15	94.23
3	134.23	38.6

1. What are the assumptions of the Empirical Bayes Credibility Theory (EBCT) Model 1?
2. Calculate the credibility premium of each risk under the assumptions of (EBCT) Model 1.

Exercise 2.

1. Let Z be the credibility factor under EBCT Model 1. What is the range of values of Z?
2. Looking at Exercise 1, the credibility premium you have calculated are essentially the credibility estimate of next years expected aggregate claims for risk i, $i = 1, 2, 3$. Explain the relationship between the value of Z and the degree of "trust" we place on $\overline{X_i}$ compared to $\overline{X}$ as an estimate of next years expected aggregate claims for risk i.

Exercise 3. The table below shows annual aggregate claim statistics for 3 risks over 4 years. Annual aggregate claims for risk i, in year j are denoted by X_{ij}

Risk, i	$\bar{X}_i = \frac{1}{4}\sum_{j=1}^{4} X_{ij}$	$s_i^2 = \frac{1}{3}\sum_{j=1}^{4}\left(X_{ij} - \bar{X}_i\right)^2$
1	2,517	4,121,280
2	7,814	7,299,175
3	2,920	3,814,001

1. Calculate the value of the credibility factor for Empirical Bayes Model 1.
2. Using the numbers calculated in 1. to illustrate your answer, describe the way in which the data affects the value of the credibility factor.

Exercise 4. The table below shows the aggregate claim amounts (in £m) for an insurance company's fire insurance portfolio for a 5-year period, together with some summary statistics.
Fill in the missing entries and calculate $E[m(\theta)]$ and $E[s^2(\theta)]$ using EBCT Model 1.

	Total claim amount Year(j)					$\bar{X}_i$	$\frac{1}{4}\sum_{j=1}^{5}(X_{ij}-\bar{X}_i)^2$
Country(i)	**1**	**2**	**3**	**4**	**5**		
1	48	53	42	50	59	50.4	39.3
2	64	71	64	73	70	68.4	17.3
3	85	54	76	65	90	74	215.5
4	44	52	69	55	71	?	?

1. Fill in the missing entries and calculate $E[m(\theta)]$ and $E[s^2(\theta)]$ using EBCT Model 1.
2. Estimate the value of var$[m(\theta)]$.
3. Calculate the credibility factor and hence calculate the EBCT premium for each of the countries in the coming year.

Exercise 5. An actuary wishes to analyse the amounts paid by a group of insurers on their respective portfolios of commercial property insurance policies using the models of Empirical Bayes Credibility Theory.

The actuary obtains the following information about the amounts of claim payments (in millions of pounds) made and the number of policies sold for each of three different insurers. The data obtained are as follows, with the notation (claim payments; number of policies):

		Year 1	**Year 2**	**Year 3**	**Year 4**
	A	14.2 ; 163	15.8 ; 189	22.7 ; 252	19.0 ; 199
Insurer	**B**	58.6 ; 4435	63.1 ; 4761	81.0 ; 5576	64.2 ; 4581
	C	123 ; 16,184	132 ; 17,443	161 ; 20,102	133 ; 18,000

Analyse the data using EBCT Model 2, and calculate the expected payout amount for Insurer B in the coming year, assuming that the expected number of policies sold for the coming year for Insurer B is 4,800.

You may use the summary statistics given below, again working in in millions of pounds. Subscripts 1, 2 and 3 refer to Insurers A, B and C respectively.

$$\sum_j P_{1j}\left(X_{1j}-\overline{X_1}\right)^2 = 0.014667$$

$$\sum_j P_{2j}\left(X_{2j}-\overline{X_2}\right)^2 = 0.006103$$

$$\sum_j P_{3j}\left(X_{3j}-\overline{X_3}\right)^2 = 0.003979$$

$$\sum_j P_{1j}\left(X_{1j}-\overline{X}\right)^2 = 5.106461$$

$$\sum_j P_{2j}\left(X_{2j}-\overline{X}\right)^2 = 0.336408$$

$$\sum_j P_{3j}\left(X_{3j}-\overline{X}\right)^2 = 0.292641$$