Vectors & Matrices

Problem Sheet 7

1. Define

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

   (i) Evaluate B^2.

   (ii) By writing A as $I_2 + B$, evaluate A^2.

   (iii) Find a formulation for the matrix A^n, for any $n \in \mathbb{N}$.

   (iv) Show that A is invertible and find its inverse.

2. Let A and B be $m \times n$ matrices. Prove $(A+B)^T = A^T + B^T$.

3. Show that the matrix

$$A = \begin{pmatrix} 2\sqrt{3} & 3 & 3\sqrt{3} \\ 0 & 6 & -2\sqrt{3} \\ -6 & \sqrt{3} & 3 \end{pmatrix}$$

   has the property that $A^T = 48A^{-1}$.

4. Let A be an $m \times n$ matrix. Prove that $A^T A$ is symmetric.

5. Find the values $x, y, z \in \mathbb{R}$ that satisfy

$$\begin{pmatrix} x & 1 & 2 \\ 0 & y & -1 \\ 0 & 0 & z \end{pmatrix} \begin{pmatrix} -1 & 8 & -5 \\ 2 & -3 & 2 \\ 3 & 0 & -4 \end{pmatrix} = \begin{pmatrix} 11 & -27 & 9 \\ 1 & -6 & 8 \\ 9 & 0 & -12 \end{pmatrix}.$$

   Once you have found these values, apply this left matrix to the column vector $\mathbf{x} = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix}$.

6. Let A be the $n \times n$ diagonal matrix

$$\begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_{nn} \end{pmatrix}.$$

   Find a condition for A to be invertible and, assuming this condition is satisfied, find its inverse.