Practice Set Reading Week

1. John who is aged 55 exactly intends to buy a reversionary annuity of £1,000 per annum payable monthly in advance to his spouse Mary, who is currently aged 52 exact. Calculate the expected present value of the policy benefits at outset. Basis: Mortality Male PMA92C20 Female PFA92C20 Interest 4% per annum effective

Answer

$1,000 \times \ddot{a}^{(12)}_{55|52} = 1,000 \times (\ddot{a}^{(12)}_{52} - \ddot{a}^{(12)}_{55:52}) = 1000 \times (\ddot{a}_{52} - \frac{11}{24} - \ddot{a}_{55:52} + \frac{11}{24})$

2 [2023]. A life assurance company issues a 20-year term, insurance policy to a life aged 40, with sum insured $\pounds 100,000$ payable immediately on death. Level premiums are payable monthly in advance throughout the term. The commissions are 10% of each premium payment (incurred at the premium payment times). Mortality follows AM92 ultimate life table with Uniform distribution of deaths (UDD) assumption between integer years. $i = 6\%$.
a) Write down an expression for the gross loss at issue random variable.
c) Write down the equation of value for this policy.
b) Calculate the gross monthly premium.

Answer
a)

$$L_0^g = PV \text{ of benefits } + \ PV \text{ of expenses} - PV \text{ of premiums}$$

Let P be the monthly premium.
PV of benefits:

$$100,000\left[\exp\left(-\delta T_{40}\right) \times 1\left(T_{40} \leq 20\right)\right]$$

- T_{40} be the future life time random variable of a life aged 40
- $\left[\exp\left(-\delta T_{40}\right) \times 1\left(T_{40} \leq n\right)\right]$ is the PV of a continuous term insurance with unit benefit.
- $1\left(T_{40} \leq 20\right)$ is an indicator function which accounts for whether the insured dies within 20 years

PV of expenses:

$$0.1P\ddot{a}^{(12)}_{\overline{min(K_{40}+1),20}|}$$

PV of premiums:

$$P\ddot{a}^{(12)}_{\overline{min(K_{40}+1),20|}}$$

Hence

$$L_0^g = 100,000\left[\exp\left(-\delta T_{40}\right) \times 1\left(T_{40} \le 20\right)\right] - 0.9P\ddot{a}^{(12)}_{\overline{min(K_{40}+1),20|}}$$

b)

The equation of value:

$$E\left(L_0^g\right) = 0$$

$$100,000\bar{A}^1_{40:\overline{20|}} = 0.9P\ddot{a}^{(12)}_{40:\overline{20|}}$$

c)

$$P = \frac{100,000\bar{A}^1_{40:\overline{20|}}}{0.9\ddot{a}^{(12)}_{40:\overline{20|}}} = 265.3716$$

We used the following calculations:

$$\bar{A}^1_{40:\overline{20|}} = \frac{i}{\delta}A^1_{40:\overline{20|}}$$

$$A^1_{40:\overline{20|}} = A_{40:\overline{20|}} - {}_{20}E_{40}$$

$${}_{20}E_{40} = v^{20}\ {}_{20}p_{40} = 0.2938$$

since ${}_{20}p_{40} = \frac{l_{60}}{l_{40}} = \frac{9287.2164}{9856.2863} = 0.942263$ and $v^{20} = \left(\frac{1}{1.06}\right)^{20}$

$$A_{40:\overline{20|}} = 0.32088$$

from the AM92 table

Hence

$$A^1_{40:\overline{20|}} = 0.32088 - 0.2938 = 0.027078$$

$$\begin{aligned}\bar{A}^1_{40:\overline{20|}} &= \frac{0.06}{\ln\left(1.06\right)}A^1_{40:\overline{20|}} \\ &= 0.027882\end{aligned}$$

$$\begin{aligned}\ddot{a}^{(12)}_{40:\overline{20|}} &\simeq \ddot{a}_{40:\overline{20|}} - \frac{12-1}{24}(1 - v^{20}\ {}_{20}p_{40}) \\ &= 11.998 - \frac{11}{24}\left(1 - 0.2938\right) \\ &= 11.67433\end{aligned}$$

3 [2023]. Shaun and Riley are independent lives, both aged 60. They purchase an insurance policy which provides £200,000 payable at the end of the year of Shaun's death, provided that Shaun dies after Riley. Annual premiums are payable in advance throughout Shaun's lifetime. You are given $\ddot{a}_{60} = 15.632$, $\ddot{a}_{70} = 11.762$ and $\ddot{a}_{60:60} = 14.090$, $\ddot{a}_{70:70} = 9.766$ and $i = 4\%$ per annum.

a) Calculate the net annual premium of this policy.

b) Calculate the net premium policy value after 10 years if only Shaun is alive.

b) Calculate the net premium policy value after 10 years if both Shaun and Riley are alive

d) Explain why the insurer cares whether both are alive or only Shaun is alive.

Answer

a)

Let P represent the annual premium.

From the Tables: $\ddot{a}_{60} = 15.632$ $\ddot{a}_{60:60} = 14.090$. Then

$A_{60} = (1 - d\ddot{a}_{60}) = \left(1 - \frac{0.04}{1.04} \times 15.632\right) = 0.398769$

$A_{60:60} = (1 - d\ddot{a}_{60:60}) = \left(1 - \frac{0.04}{1.04} \times 14.090\right) = 0.458076$

The EPV of the premiums is:

$$P\ddot{a}_{60} = 15.632P$$

The EPV of the death benefit is (initially I will use letters S,R for the lives to avoid confusion):

$$200,000A_{\overset{2}{S}:R}$$

Now, $A_S = A_{\overset{1}{S}:R} + A_{\overset{2}{S}:R}$ so $A_{\overset{2}{S}:R} = A_S - A_{\overset{1}{S}:R}$.

Replacing S and R by the age 60 and noting by symmetry that $A_{\overset{1}{60}:60} = \frac{1}{2}A_{60:60}$ we can say that $A_{\overset{2}{60}:60} = A_{60} - \frac{1}{2}A_{60:60} = 0.16973$.

The EPV of the death benefit is:

$$200,000 \times A_{\overset{2}{60}:60} = 33,946.1539$$

Hence by equivalence principle

$$P = \frac{33,946.15385}{15.632} = 2171.7810$$

b)

$\ddot{a}_{70} = 11.762$ and $\ddot{a}_{70:70} = 9.766$, hence $A_{70} = 0.5476$ and $A_{70:70} = 0.2354$

If Shaun is alive but Riley not the policy value is:

$${}_{10}V = 200,000A_{70} - P\ddot{a}_{70} = 83,980.94146$$

c)

If both are alive:

$$ {}_{10}V = 200,000\left(A_{70} - \frac{1}{2}A_{70:70}\right) - P\ddot{a}_{70} = 60,438.6337 $$

d)

The first policy value (at b) is higher than that at c) as the likelihood of paying the death benefit is higher in this case as Riley is already dead. In the second case there is still a positive probability that Riley dies after Shaun which means that no death benefit is paid. Hence, the insurer needs to build more reserves if only Shaun is alive.

4. Consider a fully discrete whole life insurance with sum insured £190,000 issued to a life aged 35. The level premiums are paid annually in advance and the payment term is 20 years.

Commissions are 7% of the first premium and 5% of the subsequent premiums. Assume the mortality follows the AM92 ultimate life table with $i = 4\%$ per annum.

a) Write down an expression for the gross loss at issue random variable

b) Calculate the annual premium.

c) Calculate the probability that the contract makes a profit

Answer

a)

Gross future loss at issue:

$$ L_0^n = 190,000v^{K_{35}+1} + 0.05\ddot{a}_{\overline{\min(K_{35}+1,20)}|} + 0.02P - P\ddot{a}_{\overline{\min(K_{35}+1,20)}|} $$

Equivalence principle:

$$ E\left(L_0^n\right) = 190,000A_{35} - 0.95P\ddot{a}_{35:\overline{20}|} + 0.02P = 0 $$

b)

$\ddot{a}_{35:\overline{20}|} = \ddot{a}_{35} - \; v^{20} \; {}_{20}p_{35}\ddot{a}_{55}$

${}_{20}p_{35} = \frac{l_{55}}{l_{35}} = \frac{9557.9179}{9894.4299} = 0.96699$

$\ddot{a}_{35:\overline{20}|} = 21.003 - (\frac{1}{1.04})^{20} \times 0.96699 \times 15.873 = 14.03215$

$$ \begin{aligned} P &= \frac{190,000A_{35}}{0.95\ddot{a}_{35:\overline{20}|} - 0.02} \\ &= \frac{190,000 \times 0.19219}{0.95 \times 14.03215 - 0.02} \\ &= 2743.3972 \end{aligned} $$

c)
Did the policy made a profit at death?
At death:

- Benefit outgo: 190,000
- Income: premiums for 20 years only - bring them to the time of death?

$$\underbrace{\ddot{a}_{35:\overline{20}|} \times P\,(1.04)^{20} \times}_{FV \text{ at the end of 20 years}} (1.04)^{n-20} > 190,000$$

$$\ddot{a}_{35:\overline{20}|} \times P\,(1.04)^{20} \times (1.04)^{n-20} = \frac{190,000}{\ddot{a}_{35:\overline{20}|} \times P\,(1.04)^{20}}$$

$$\begin{aligned} n &= \left\lfloor \ln\left(\frac{190,000}{\ddot{a}_{35:\overline{20}|} \times P\,(1.04)^{20}}\right) / \ln(1.04) + 20 \right\rfloor \\ n &= 58 \end{aligned}$$

We want the smallest integer such that the accumulations of premiums to time n exceed the sum insured. The life needs to live 57 years or more for profit to be made, hence probability of profit ${}_{57}p_{35} = 11.34\%$

5 [2023]. A life assurance company sells a life annuity which pays out £1,000 per year to one individual aged 60 for £11,280 (one payment at the signing of the contract). The annuity is paid annually in arrears. The valuation basis for this question is as follows: Mortality follows AM92 ultimate, interest: 6% per annum.

a) Express the profit random variable at the issuance of this policy.

b) Calculate the expected present value of the life insurer's profit.

c) Calculate the standard deviation of the present value of the life insurer's profit.

d) Calculate the probability that the present value of the life insurer's profit on this contract will be positive.

e) Do you have any concerns about issuing this policy, from a profit viewpoint? Briefly explain your answer.

d) Suppose the life annuity was paid annually in advance. Without doing any more calculations, would the standard deviation of the present value of the profit be less than, equal to or greater than the standard deviation calculated in (b)? Explain your answer.

Answer

a)

$$\begin{aligned} P &= \text{Income - Benefits outgo} \\ &= 11280 - 1000 a_{\overline{K_{60}}|} \end{aligned}$$

b)

From the table $\ddot{a}_{60} = 11.891$ hence $a_{60} = 10.891$

$$\begin{aligned} EPV(P) &= 11280 - 1000 a_{60} \\ &= 11280 - 1000 \times 10.891 = 389 \end{aligned}$$

c)

$$\begin{aligned} Var(P) &= 1000^2 \times Var\left(a_{\overline{K_{60}}|}\right) \\ &= 1000^2 \times Var\left(\frac{1 - v^{K_{60}}}{i}\right) \\ &= 1000^2 \times \frac{1}{(iv)^2} Var\left(v^{K_{60}+1}\right) \\ &= 1000^2 \times \frac{1}{d^2}\left({}^2A_{60} - A_{60}^2\right) \\ &= 1000^2 \times \frac{0.14098 - (0.32692)^2}{(1 - 1.06^{-1})^2} \end{aligned}$$

$$StDev(P) = \sqrt{Var(P)} = 3263$$

d)

$$\begin{aligned} \Pr(P > 0) &= \Pr\left(1000\left(11.28 - a_{\overline{K_{60}}|}\right) > 0\right) \\ &= \Pr\left(11.28 - a_{\overline{K_{60}}|} > 0\right) \\ &= \Pr\left(v^{\overline{K_{60}}|} > 1 - 11.28i\right) \\ &= \Pr\left((1+i)^{-\overline{K_{60}}|} > 1 - 11.28i\right) \\ &= \Pr\left(K_{60} < -\frac{\ln(1 - 11.28i)}{\ln(1+i)}\right) \\ &= \Pr(K_{60} < 19.384) \\ &= \Pr(K_{60} \leq 19) \\ &= \Pr(K_{60} + 1 \leq 20) \\ &= {}_{20}q_{60} \\ &= 1 - {}_{20}p_{60} = 1 - \frac{l_{80}}{l_{60}} \\ &= 1 - \frac{5266.4604}{9287.2164} = 0.4329 \end{aligned}$$

d)

As calculated in part (c), there is a 56.71% chance of a loss.

The maximum possible loss is £1,000 annuity in arrears for life: PV is $= 1000\left(\frac{1}{1-\frac{1}{1+i}} - 1\right) = \frac{1000}{i} =$ £16,666.67 which yaking into account the premium £11,280 gives a loss of £5,386.67. The maximum possible profit is £11280, corresponding to the person dying before age 61. Therefore, the present value of the profit lies in the interval $(-5386.67, 11280]$.

The expected present value of the profit is positive at £389. However, the standard deviation is almost ten times as large, at £3263. This suggests that the distribution of the present value of the profit is spread out.

e) It would not change at all. The only difference would be a payment at age 60 with probability 1, which would not affect the variance (or standard deviation).

6 [2023]. On 1 January 2017, an insurer issued whole life assurances to lives then aged exactly 65. The number of policies in force on 1 January 2022 was 1,900, the number in force on 1 January 2023 was 1,867. The sum insured was £100,000 payable at the end of the year of death. The level premiums are payable annually in advance for the whole of life. Assume that death is the only cause of policy termination, and that the insurer holds net premium reserves for these contracts. Mortality follows AM92 ultimate life table and interest is 4% per annum.

a) Calculate the net premium for these policies

b) Calculate the death strain at risk for the policy in the year commencing on 1 January 2022

c) Calculate the mortality profit for the policy in the year commencing on 1 January 2022.

Answer

a)

From the equivalence principle:

$$\begin{aligned} P &= \frac{100,000\ A_{65}}{\ddot{a}_{65}} = \\ &\quad \frac{100,000 A_{65}}{\ddot{a}_{65}} \\ &= 100,000 \times \frac{0.52786}{12.276} \\ &\quad 4,299.935 \end{aligned}$$

b)

For calculating the profit at the end of 2022 we need to calculate the reserve at the end on 2022

$$
\begin{aligned}
{}_6V &= 100,000A_{71} - P\ddot{a}_{71} \\
&= 100,000 \times 0.61548 - 4,299.935 \times 9.998 \\
&= 18,557.25
\end{aligned}
$$

$$DSAR = 100,000 - 18,557.25 = 81,442.75$$

c)

$$
\begin{aligned}
EDS &= 1,900 \times q_{70} \times DSAR \\
1,900 \times 0.024783 \times 81,442.75 &= 3,834,972
\end{aligned}
$$

During the policy year, 33 people died, so the actual death strain was:

$$ADS = 33 \times DSAR = 2,687,611$$

This gives a mortality profit of $EDS - ADS = 1,147,341$