Vectors & Matrices

Problem Sheet 6

1. For the following matrices,

$$A = \begin{pmatrix} 5 & -2 & 1 \\ -1 & 3 & -3 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 8 \\ -4 & 2 \\ -7 & 2 \end{pmatrix}, \qquad C = \begin{pmatrix} -3 & -2 \\ 7 & 1 \\ 1 & -5 \end{pmatrix},$$

compute the matrices AB, AC, $B + C$, $A(B + C)$ and $2A(C + B)$.

2. Prove that for any $m \times n$ matrices A and B, and any real value $\alpha \in \mathbb{R}$,

$$\alpha(A + B) = \alpha A + \alpha B \, .$$

3. Consider the matrix A, given by

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} .$$

Evaluate the matrix A^{2024}.

4. Prove that there exists only one matrix I_n such that for all $n \times n$ matrices A,

$$I_n A = A I_n = A \, .$$

(In other words, prove that identity matrices for $n \times n$ systems are unique.)

5. Consider the matrices

$$A = \begin{pmatrix} 2 & 1 \\ 6 & z \end{pmatrix}, \qquad B = \begin{pmatrix} z & -1 \\ -6 & 2 \end{pmatrix},$$

where $z \in \mathbb{R}$ is a scalar.

   (i) Evaluate the products AB and BA.

   (ii) Using this product, find the inverse of A.

   (iii) For what value of z does an inverse of A not exist?

6. Prove that for any natural $n > 1$, the matrix

$$A = (1)_{n \times n}$$

(that is, the $n \times n$ matrix consisting only of ones) is not invertible.