Matrices

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Theorem

Let A, B and C be matrices of the same size, and let α and β be scalars. Then:

(a) $A + B = B + A$;

(b) $A + (B + C) = (A + B) + C$;

(c) $A + O = A$;

(d) $A + (-A) = O$, where $-A = (-1)A$;

(e) $\alpha(A + B) = \alpha A + \alpha B$;

(f) $(\alpha + \beta)A = \alpha A + \beta A$;

(g) $(\alpha\beta)A = \alpha(\beta A)$;

(h) $1A = A$.

Proof

Examples and conclusion

Matrix multiplication

Definition

If $A = (a_{ij})$ is an $m \times n$ matrix and $B = (b_{ij})$ is an $n \times p$ matrix then the **product** AB of A and B is the $m \times p$ matrix $C = (c_{ij})$ with

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} \, .$$

Definition

An **identity matrix** I is a square matrix with 1's on the diagonal and zeros elsewhere. If we want to emphasise its size we write I_n for the $n \times n$ identity matrix.

Theorem

Let $A = (a_{ij})_{m\times n}$, $B = (b_{ij})_{m\times n}$, $C = (c_{ij})_{n\times p}$ and $D = (d_{ij})_{n\times p}$ be matrices and $\alpha \in \mathbb{R}$. Then,

(a) $(A + B)C = AC + BC$ and $A(C + D) = AC + AD$;

(b) $\alpha(AC) = (\alpha A)C = A(\alpha C)$;

(c) $I_m A = A I_n = A$;

Let $X = (x_{ij})_{m\times n}$, $Y = (y_{ij})_{n\times p}$ and $Z = (z_{ij})_{p\times q}$. Then,

(d) $(XY)Z = X(YZ)$.

Proof

Proof

In general $AB \neq BA$

Inverse matrix