MTH5114 Linear Programming and Game Theory, Spring 2024

Week 5 Coursework Questions **Viresh Patel**

These exercises should be completed individually and submitted (together with those of weeks 4 and 6) via the course QMPlus page by **9am on Monday, 11 March**.

Make sure you clearly write your **name** and **student ID** number at the top of your submission:.

Solve the following linear program using the simplex algorithm. You should give the initial tableau and each further tableau produced during the execution of the algorithm. If the program has an optimal solution, give this solution and state its objective value. If it does not have an optimal solution, say why.

You should indicate the highlighted row and columns in each pivot step as well as the row operations you carry out. This is in order to gain credit even if the final answer is incorrect.

1.
$$\begin{aligned} \text{maximize} \quad & 2x_1 + 3x_2 + 5x_3 + x_4 \\ \text{subject to} \quad & x_1 + x_2 + 2x_3 + x_4 \leq 2, \\ & 3x_2 + 3x_3 + 3x_4 \leq 6, \\ & 3x_1 + 2x_2 + 2x_3 + x_4 \leq 7, \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

2. Suppose that we are carrying out the simplex algorithm on a linear program in standard inequality form (with 3 variables and 4 constraints) and suppose that we have reached a point where we have obtained the following tableau. Apply one more pivot operation, indicating the highlighted row and column and the row operations you carry out. What can you conclude from your updated tableau?

	x_1	x_2	x_3	s_1	s_2	s_3	s_4	
s_1	-2	0	1	1	0	0	0	3
s_2	3	0	-2	0	1	2	0	6
x_2	1	1	-3	0	0	1	0	2
s_4	-3	0	2	0	0	-1	1	4
$-z$	-2	0	11	0	0	-4	0	-8