MTH5114 **Linear Programming and Games, Spring 2024**

Week 4 Seminar Questions **Viresh Patel**

Question (similar to past exam questions): A set of points $P \subseteq \mathbb{R}^n$ is called *convex* if it satisfies the property that if $\mathbf{x} \in P$ and $\mathbf{y} \in P$ then $\lambda\mathbf{x} + (1-\lambda)\mathbf{y} \in P$ for all $\lambda \in [0, 1]$ (i.e. if $\mathbf{x} \in P$ and $\mathbf{y} \in P$ then every convex combination of $\mathbf{x}$ and $\mathbf{y}$ is in P).

Consider an arbitrary linear program in standard equation form. Show that the set of all feasible solutions to this linear program must be a convex set.

Discussion Questions:

1. Look again at the warm up question from the week 3 seminar questions and its solution. There, you were asked to sketch the feasible region of the following linear program.

$$\begin{aligned} \text{maximize} \quad & 2x_1 + x_2 \\ \text{subject to} \quad & 2x_1 + 3x_2 \leq 12, \\ & 4x_1 + 2x_2 \leq 12, \\ & x_1, x_2 \geq 0 \end{aligned}$$

   (a) Transform this linear program into standard equation form and write down the corresponding constraint matrix A and vectors, $\mathbf{b}$ and $\mathbf{c}$.

   (b) For each of the following feasible solutions of the original linear program, find the corresponding feasible solutions of the linear program in standard inequality form and check which of them are basic feasible solutions **(using the definition of basic feasible solution)**. [It will be useful to recap the different ways in which you can show that a set of vectors is linearly independent or linearly dependent by looking at the week 1 seminar questions and/or notes from Linear Algebra I.]

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \qquad \mathbf{v}_2 = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \qquad \mathbf{v}_3 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \qquad \mathbf{v}_4 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

2. Modify the linear program in Q1 above by adding one more inequality constraint in such a way that there is a basic feasible solution (of the corresponding standard equation form) that has three zero entries. [There are many possible solutions here.]