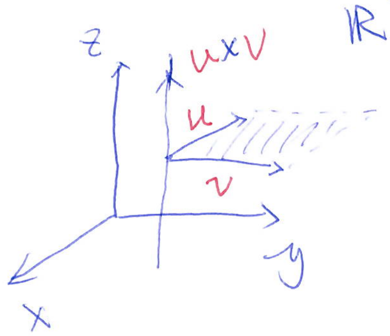
last lecture:

Vector product

$|u \times v| = |u||v| \sin\theta$

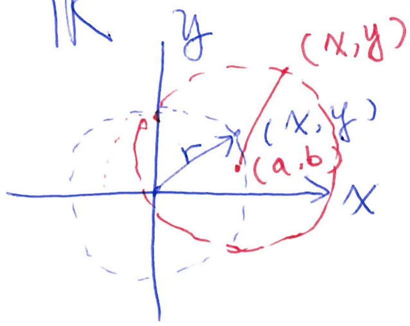
eg: $\mathbb{R}^3$



Today:

5.7 Equation of the circle in $\mathbb{R}^2$ and sphere in R^3. R^n

$\mathbb{R}^2$



$$x^2 + y^2 = r^2$$
$$\Updownarrow$$
$$|p|^2 = r^2$$
$$\Updownarrow$$
$$|p| = r$$

r: radius

p = position vector $\begin{pmatrix} x \\ y \end{pmatrix}$

in $\mathbb{R}^n$, $p = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$

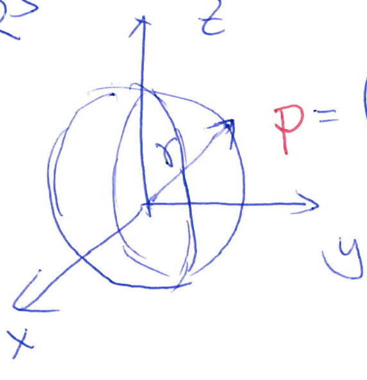
$|p| = \sqrt{\sum_{i=1}^{n} x_i^2}$

circle:

if moving the circle around point (a, b)

$$(x-a)^2 + (y-b)^2 = r^2$$

$\mathbb{R}^3$



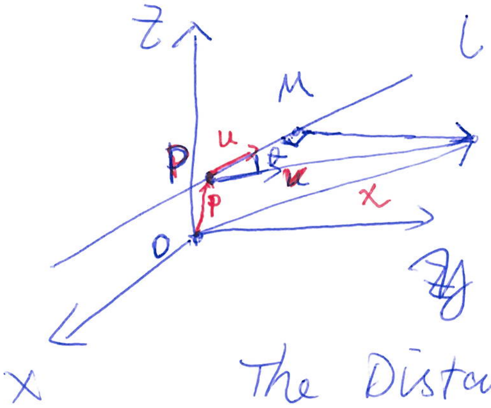
$p = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$x^2 + y^2 + z^2 = r^2$$
$$\Updownarrow$$
$$|p|^2 = r^2$$
$$\Updownarrow$$
$$|p| = r$$

$\mathbb{R}^n$: We can define sphere in any space dimension by using the length of the position vector $P. = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix}$

$$S^n = \{ (x_1, x_2, \cdots, x_n) \in \mathbb{R}^n : |P| = r \}$$

5.8. Distance from a point to a line



line l.

$r = p + \lambda u$ (Vector equation of a line)

Point X has the position vector x

The Distance from Point X to line l, means we need to find a point M, which is closest to Poin X and $|\vec{MX}|$ is the distance between X and l.

$$|\vec{MX}| = \underline{\underline{|\vec{PX}|}} \cdot \sin\theta = |v| \sin\theta = \frac{|u \times v|}{|u|}$$

$$\vec{PX} = v = x - p = \frac{|u \times (x-p)|}{|u|}$$

$$= \vec{OX} - \vec{OP}$$

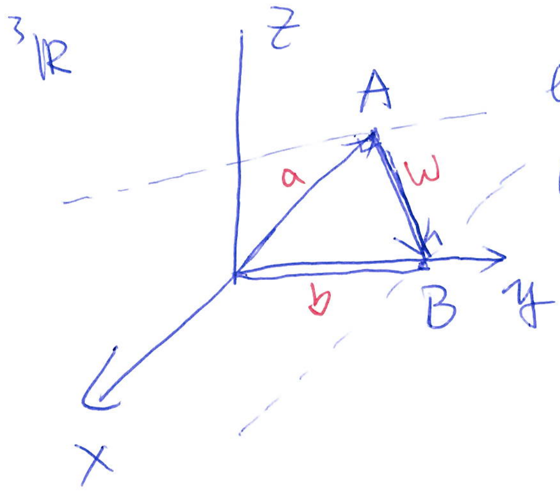
$$= \vec{OX} + \vec{PO}$$

$$= \vec{PX}$$

$|u \times v| = |u| \cdot |v| \cdot \sin\theta$

2

5.9. Distance between two lines



$l_1 \quad r = \underline{\underline{p}} + \lambda \underline{\underline{u}}$

$l_2 \quad \cancel{r}\; r = \underline{\underline{q}} + \mu \underline{\underline{v}}$

Choosing point A and B. the distance between line l_1 and line l_2, means we should minimise $|\overrightarrow{AB}|$. To fulfil this, $\overrightarrow{AB}$ should be orthogonal to line l_1 and line l_2. $\Leftrightarrow$ $\overrightarrow{AB}$ should be orthogonal to both u and v.

$\overrightarrow{AB} = \underline{w = \alpha(u \times v)}$

check proposition 5.5.3.
the vector product $u \times v$ is orthogonal to both u and v

$w = b - a = \overbrace{q + \mu v}^{B \text{ on line } l_2} - \overbrace{(p + \lambda u)}^{A \text{ on line } l_1}$

$$w = \alpha(u \times v)$$
$$w = q + \mu v - p - \lambda u$$

$$\alpha(u \times v)\cdot(u \times v) = (q + \mu v - p - \lambda u)\cdot(u \times v)$$

$$= q\cdot(u \times v) + \underbrace{\cancel{\mu v\cdot(u \times v)}}_{\text{orthogonal}} - p(u \times v) - \underbrace{\cancel{\lambda u\cdot(u \times v)}}_{\text{orthogonal}}$$

$$w\cdot(u \times v) = (q - p)(u \times v)$$

$$|w| = \frac{|(q-p)(u \times v)|}{|u \times v|}$$

5.10. Intersections of Planes.

Catesian equation for a plane in $\mathbb{R}^3$

$$ax+by+cz=d$$

position vector $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ of point (x, y, z).

Now let say we have k planes in $\mathbb{R}^3$

$$\begin{cases} a_1x+b_1y+c_1z=d_1 \\ a_2x+b_2y+c_2z=d_2 \\ \vdots \\ a_kx+b_ky+c_kz=d_k \end{cases}$$

The intersect of k planes defined above refers to a common solution of these k equations above.

5.11. Intersections of other objects

intersection between

- a plane and line $r = p + \lambda u$

  - line $\begin{cases} x = p_1 + \lambda u_1 \\ y = p_2 + \lambda u_2 \\ z = p_3 + \lambda u_3 \end{cases}$

  - plane $ax + by + cz = d$

Interesection: $a(p_1 + \lambda u_1) + b(p_2 + \lambda u_2) + c(p_3 + \lambda u_3) = d$