1. Next year, the economy will be in recession with probability 0.3, experience normal growth with probability 0.4, or expansion with probability 0.3. There are only two stocks available to trade in this economy (stock MJ and stock OBN). The stocks' annual returns in each of the states of the world are according to the following table:

Stocks	Recession	Normal Growth	Expansion
MJ	6%	12%	14%
OBN	15%	5%	10%

a) Show whether you can rank the stocks by the criterion of first order stochastic dominance. If yes, which stock first order stochastically dominates the other?
b) Show whether you can rank the stocks by the criterion of second order stochastic dominance. If yes, which stock second order stochastically dominates the other?
c) Calculate the expected annual return of each stock.
d) Calculate the annual variance and standard deviation of each stock.
e) Calculate the covariance and correlation coefficient between the two stocks.

Answers

a)

Stocks	Recession	Normal Growth	Expansion
MJ	6% with prob 0.3	12% with prob 0.4	14% with prob 0.3
OBN	15% with prob 0.3	5% with prob 0.4	10% with prob 0.3

Returns	MJ pdf	MJ cdf	OBN pdf	OBN cdf
5%	0	0 <	0.4	0.4
6%	0.3	0.3 <	0	0.4
10%	0	0.3 <	0.3	0.7
12%	0.4	0.7 =	0	0.7
14%	0.3	1 >	0	0.7
15%	0	1 =	0.3	1

In order to have one asset first order dominating another we need at every possible return, the cdf of returns of one asset to be less or equal to cdf of returns of the other asset with at least one strict inequality. In this case we see that we cannot first order dominance rank these assets as cdf of MJ smaller or equal to cdf of OBN for all returns apart from the case when return is 14% when it is the opposite.

b)

Returns	MJ cdf	MJ sum cdf	OBN cdf	OBN sum cdf
5%	0	0 <	0.4	0.4
6%	0.3	0.3 <	0.4	0.8
10%	0.3	0.6 <	0.7	1.5
12%	0.7	1 <	0.7	2.2
14%	1	2 <	0.7	2.9
15%	1	3 <	1	3.9

Hence we can rank and MJ second order stochastic dominates OBN.

c)

$$E(R_{MJ}) = 0.06 \times 0.3 + 0.12 \times 0.4 + 0.14 \times 0.3 = 0.108 = 10.8\%$$
$$E(R_{OBN}) = 0.15 \times 0.3 + 0.05 \times 0.4 + 0.10 \times 0.3 = 0.095 = 9.5\%$$

d)

$$Var(R_{MJ}) = (0.06 - 0.108)^2 \times 0.3 + (0.12 - 0.108)^2 \times 0.4 + (0.14 - 0.108)^2 \times 0.3 = 0.001056$$
$$Stdv(R_{MJ}) = 0.032$$

$$Var(R_{OBN}) = (0.15 - 0.095)^2 \times 0.3 + (0.05 - 0.095)^2 \times 0.4 + (0.10 - 0.095)^2 \times 0.3 = 0.001725$$
$$Stdv(R_{OBN}) = 0.042$$

e)

$$Cov(R_{MJ}, R_{OBN}) = (0.06 - 0.108) \times (0.15 - 0.095) \times 0.3 + (0.12 - 0.108) \times (0.05 - 0.095) \times 0.4 + (0.14 - 0.108) \times (0.1 - 0.095) \times 0.3 = -0.00096$$
$$Corr(R_{MJ}, R_{OBN}) = -\frac{0.00096}{0.042 \times 0.032} = -0.711$$