Vectors & Matrices

Problem Sheet 2

1. Consider the line ℓ that goes through the point $P = (3, -1, 2)$ in the direction of the vector $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$.

   (i) Show that the point $Q = (24, -43, 65)$ lies on the line ℓ.

   (ii) Does the point $R = (1, 3, -7)$ lie on ℓ?

   (iii) The point $S = (14, -23, z)$ lies on ℓ. What is the value of z?

2. Let $\mathbf{p}$ and $\mathbf{q}$ be the position vectors of the points P and Q (respectively).

   In general, which of the following sets would **not** define the straight line through points P and Q?

   - $\{\mathbf{p} + \lambda(\mathbf{q} - \mathbf{p}) : \lambda \in \mathbb{R}\}$
   - $\{\mathbf{p} + 2\lambda(\mathbf{q} - \mathbf{p}) : \lambda \in \mathbb{R}\}$
   - $\{\mathbf{q} + \lambda(\mathbf{q} - \mathbf{p}) : \lambda \in \mathbb{R}\}$
   - $\{2\mathbf{p} + \lambda(\mathbf{q} - \mathbf{p}) : \lambda \in \mathbb{R}\}$
   - $\{\mathbf{p} + \lambda(\mathbf{p} - \mathbf{q}) : \lambda \in \mathbb{R}\}$

   Give a condition on $\mathbf{p}$ that would mean that this set **does** define the line through P and Q.

3. Let $\mathbf{u} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$.

   Show that the set $\{\lambda\mathbf{u} + \mu\mathbf{v} : \lambda, \mu \in \mathbb{R}\}$ is a sub-vector space of $\mathbb{R}^3$

4. Let $\mathbf{u} = \begin{pmatrix} 4 \\ -1 \\ 4 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ -9 \\ 2 \end{pmatrix}$.

   (i) Find the value of $\mathbf{u} \cdot \mathbf{v}$.

   (ii) The vector $\mathbf{w}$ is given by $\begin{pmatrix} 2 \\ 1 \\ z \end{pmatrix}$, where $z \in \mathbb{R}$ is some undetermined parameter.

   Find the value of z that would make the vector $\mathbf{u}$ orthogonal to the vector $\mathbf{v} + \mathbf{w}$.