Recap Quiz (3 min)

Consider the following LP.

$$\begin{aligned} \text{maximise} \quad & 2x_1 - 3x_2 + 3x_3 \\ \text{subject to} \quad & x_1 + x_2 = 3 \\ & x_1 + 3x_2 \leq 6 \\ & x_1 \geq 0,\ x_2 \leq 0,\ x_3 \text{ unrestricted.} \end{aligned}$$

1) Label the (a) goal
   (b) objective function
   (c) constraints
   (d) sign restrictions

2) If we transform this LP into standard inequality form
   (a) how many variables will it have?
   (b) how many constraints will it have?

3) Write down (in terms of matrices) what a general linear program in standard inequality form looks like.

Task

maximise $x_1 + x_2$

subject to
$$-x_1 + 2x_2 \leq 3$$
$$-2x_1 + x_2 \geq -6$$
$$x_1, x_2 \geq 0$$

Which are feasible solutions?

Which three are not optimal solutions?

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 100 \end{pmatrix}, \quad \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

Example 1.1. A university student is planning her daily food budget. Based on the British Nutrition Foundation's guidelines for an average female of her age she should consume the following daily amounts of vitamins:

	Vitamin	mg/day
B1	Thiamin	0.8
B2	Riboflavin	1.1
B3	Niacin	13
	Vitamin C	35

After doing some research, she finds the following cost, calories, and vitamins (in mg) per serving of several basic foods:

Food	Cost	Thiamin	Riboflavin	Niacin	Vitamin C
Bread	£0.25	0.1	0.1	1.3	0.0
Beans	£0.60	0.2	0.1	1.1	0.0
Cheese	£0.85	0.0	0.5	0.1	0.0
Eggs	£1.00	0.2	1.2	0.2	0.0
Oranges	£0.80	0.2	0.1	0.5	95.8
Potatoes	£0.50	0.2	0.1	4.2	28.7

How can the student meet her daily requirements as cheaply as possible?

Mathematical Program

Goal

Objective Function

$$
\begin{array}{ll}
\text{minimise} & 0.25x_1 + 0.60x_2 + 0.85x_3 + 1.00x_4 + 0.80x_5 + 0.50x_6 \\
\text{subject to} & 0.1x_1 + 0.2x_2 + 0.0x_3 + 0.2x_4 + 0.2x_5 + 0.2x_6 \geq 0.8 \\
 & 0.1x_1 + 0.1x_2 + 0.5x_3 + 1.2x_4 + 0.1x_5 + 0.1x_6 \geq 1.1 \\
 & 1.3x_1 + 1.1x_2 + 0.1x_3 + 0.2x_4 + 0.5x_5 + 4.2x_6 \geq 13 \\
 & 0.0x_1 + 0.0x_2 + 0.0x_3 + 0.0x_4 + 95.8x_5 + 28.7x_6 \geq 35 \\
 & x_1, x_2, x_3, x_4, x_5, x_6 \geq 0
\end{array}
$$

Constraints

Variables

Restrictions

Example 2.1. A factory makes 2 different parts (say, part X and part Y). Their plant has 4 separate processes in place: there are two older processes (say, process 1 and 2) that produce parts X and Y directly, as well as two different integrated processes for producing both X and Y simultaneously. The 4 processes can be run simultaneously, but require labour, raw metal, and electricity. The hourly inputs and outputs for each process are as follows:

	Outputs		Inputs		
Process	X	Y	Metal	Electricity	Labour
1	4	0	100 kg	800 kWh	16 hrs
2	0	1	70 kg	600 kWh	16 hrs
3	3	1	120 kg	2000 kWh	50 hrs
4	6	3	270 kg	4000 kWh	48 hrs

~~In a typical day~~, the plant has an available stock of 6000 kg of metal, and the has budgeted 100000 kWh of power usage, 1000 hours of labour. Suppose that each part X sells for £1000 and each part Y sells for £1800. How should production be scheduled to maximise daily revenue?

Task: What choices have to be made 1 ?

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Example 2.2. Suppose that our factory in Example 2.1 wants to determine its daily operating budget. It has determined that there is daily demand for 120 parts X and 50 parts Y. Suppose now that there is an unlimited amount of metal, electricity, and labour available, but the cost of metal is £5 per kg, the cost of electricity is £0.15 per kWh, and the cost of labour is £20 per hour. How can it schedule production to meet its demand as cheaply as possible?

	Outputs		Inputs		
Process	X	Y	Metal	Electricity	Labour
1	4	0	100 kg	800 kWh	16 hrs
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Example 2.3. Suppose that our factory in the previous 2 examples now wants to find a production schedule that maximises its daily *profits* defined as revenue minus costs. How can this be done? You should assume that any amount of resources are available, and that any number of parts can be sold (where the prices are given as in the previous 2 examples).

	Outputs		Inputs		
Process	X	Y	Metal	Electricity	Labour
1	4	0	100 kg	800 kWh	16 hrs
2	0	1	70 kg	600 kWh	16 hrs
3	3	1	120 kg	2000 kWh	50 hrs
4	6	3	270 kg	4000 kWh	48 hrs

metal costs £5 per kg
electric £0.15 per kWh
labour £20 per hour

Variables
$p_1, \ldots, p_4, x, y, m, e, l$

Example 2.4. A medical testing company is making diagnostic tests. Each test requires a combination of 3 different reagents:

Test	Reagents Needed		
	1	2	3
Standard	0.9 ml	1.2 ml	
Rapid	1.5 ml		1.0 ml

Each reagent can be synthesised from a combination of more basic chemicals (let's call them chemical A, B, and C), which requires some amount of laboratory time. Additionally, these reagents can be purchased from a supplier for a listed price, and any extra reagent that the company produces can also be sold to the supplier for this price. The relevant materials and costs are summarised in the following table:

Reagent	Chemicals Needed			Lab time	Price
	A	B	C	to synthesise	
1	1.0 ml	0.3 ml	1.5 ml	0.02 hrs/ml	£2.40/ml
2	0.5 ml	0.2 ml	1.0 ml	0.04 hrs/ml	£1.60/ml
3	0.2 ml	1.8 ml	0.6 ml	0.05 hrs/ml	£1.50/ml

The company has taken on a contract to produce 1000 standard tests and 2300 rapid tests. It has 100 hours of laboratory time available at a cost of £150 per hour, 1100ml of chemical A, 1250ml of chemical B, and 1800ml of chemical C available. Additionally, it can purchase and sell an unlimited amount of each reagent for the specified price. Find a production plan that fulfils the contract at the lowest net cost, taking into account any money recovered by the sale of excess reagents.

Test	Reagents Needed		
	1	2	3
Standard	0.9 ml	1.2 ml	
Rapid	1.5 ml		1.0 ml

Reagent	Chemicals Needed			Lab time to synthesise	Price
	A	B	C		
1	1.0 ml	0.3 ml	1.5 ml	0.02 hrs/ml	£2.40/ml
2	0.5 ml	0.2 ml	1.0 ml	0.04 hrs/ml	£1.60/ml
3	0.2 ml	1.8 ml	0.6 ml	0.05 hrs/ml	£1.50/ml

Transportation problem

Example 2.5. A mining company has 2 mines, where ore is extracted, and 3 warehouses, where ore is stored. Currently, there is 45Mg of ore divided amongst the mining locations. In order to prepare it for sale, this ore needs to be distributed to the warehouses. The amount of ore available at each mine, and the amount of ore required at each warehouse is as follows:

	Ore Available
Mine 1	19
Mine 2	26

	Ore Required
Warehouse 1	14
Warehouse 2	11
Warehouse 3	20

Due to different distances and shipping methods, the cost (in thousands of pounds) to ship 1 Mg depends on where it is being shipped from and where it is being shipped to, as follows:

	Warehouse 1	Warehouse 2	Warehouse 3
Mine 1	10	5	12
Mine 2	9	7	13

Suppose that these costs scale up linearly in the amount of ore that is shipped (for example, it costs $3 \cdot 10$ to ship 3Mg of ore from Mine 1 to Warehouse 1. How should we send the ore from the mines to the warehouses to minimise the overall transportation cost?

Transportation problems

Typically

- Move resources from some sources to some destinations
- Different cost of moving resource from source i to destination j

  Want to minimise total cost.

- some amount of resource available at each source

  (get one constraint for each source)

- some target amount of resource needed at each destination

  (get one constraint for each destination).

Ask following as question

An ebay trader has a £100 available and there are 3 types of items on sale today (with unlimited availability)
Tomorrow when the sale ends the trader will sell the items at a higher price.

The trader has £100.

Item	Bying Price	Selling price
A	90	180
B	20	38
C	60	117

Which items should he buy today. to maximize profit.