Vectors & Matrices

Problem Sheet 1

1. Let $A = (1, 1, 1)$ and $B = (2, 2, 2)$ be points in $\mathbb{R}^3$.

   Which of the following vectors is **not** equivalent to $\overrightarrow{OA}$?

   - $\overrightarrow{AB}$
   - $\mathbf{i} + \mathbf{j} + \mathbf{k}$
   - $\overrightarrow{OB}$
   - $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
   - $\overrightarrow{OB} + \overrightarrow{BA}$

2. Consider the vectors $\overrightarrow{AB} = \mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$, $\overrightarrow{BC} = 3\mathbf{i} + \mathbf{j} + 4\mathbf{k}$, $\overrightarrow{CD} = -\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$.

   Find the coordinates of the point E such that $\overrightarrow{OE} = \overrightarrow{AD}$.

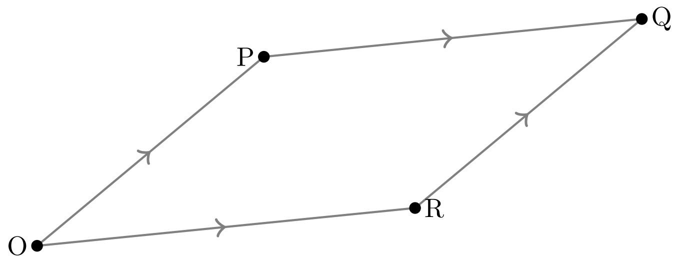
3. For any vector $\mathbf{v}$, we define the negation of $\mathbf{v}$ by:

$$-\mathbf{v} = -1 \cdot \mathbf{v}$$

   (i) Prove that for any point $A \in \mathbb{R}^3$, we have $-\overrightarrow{OA} = \overrightarrow{AO}$.

   (ii) By using the properties of vectors listed in Proposition 3.2.1, generalise part (i) to show that for any points $A, B \in \mathbb{R}^3$, we have $-\overrightarrow{AB} = \overrightarrow{BA}$.

4. Consider the following diagram of the parallelogram $OPQR$:



   Let $\mathbf{p} = \overrightarrow{OP}$, $\mathbf{q} = \overrightarrow{OQ}$ be the position vectors of the points P and Q (respectively).

   Express each of the following vectors in terms of $\mathbf{p}$ and $\mathbf{q}$:

   (i) $\overrightarrow{QO}$, (ii) $\overrightarrow{OR}$, (iii) $\overrightarrow{PQ}$, (iv) $\overrightarrow{QR}$, (v) $\overrightarrow{RP}$

5. Prove that for any vector $\mathbf{v}$ and any scalar $\lambda \in \mathbb{R}$, we have $|\lambda \mathbf{v}| = |\lambda||\mathbf{v}|$.

   How can we interpret this result geometrically?