METRIC SPACES AND TOPOLOGY

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Topics:

(1) The notion of a metric space, axioms M1, M2, M3.
(2) Examples of metrics: Euclidean metric, d_p for $p \in [1, \infty)$.
(3) Cauchy inequality $a \cdot b \leq ||a|| \cdot ||b||$.
(4) Metric d_p on $\mathbf{R}^m$.
(5) Young inequality $ab \leq \frac{a^p}{p} + \frac{b^q}{q}$ where $\frac{1}{p} + \frac{1}{q} = 1$ and its proof.
(6) Hölder inequality $a \cdot b \leq ||a||_p \cdot ||b||_q$ and its proof.
(7) Revision: the notions related to supremum and infimum, bounded and unbounded subsets of $\mathbf{R}$.
(8) Minkowski inequality $||a + b||_p \leq ||a||_p + ||b||_p$ and its proof.
(9) Metric d_∞ on $\mathbf{R}^m$ and $\lim d_p = d_\infty$ when $p \to \infty$.
(10) Balls and spheres in metric spaces.
(11) Example: the ball in d_1 metric.
(12) Example: the ball in d_∞ metric.
(13) Continuous maps between metric spaces, homeomorphism.
(14) Isometry. Isometries of the plane: reflections, rotations, parallel translations.
(15) Open sets in metric spaces
(16) Closed sets in metric spaces; closure of a set, limit points.
(17) Convergent sequences
(18) Dense subsets; examples
(19) Characterisation of open subsets of $\mathbf{R}$.
(20) The Cantor set.
(21) Infinite dimensional metric spaces: $C[a, b]$, l_2, l_p (where $p \in [1, \infty)$), l_∞.
(22) Cauchy sequences. Any convergent sequence is Cauchy.
(23) Complete metric spaces.
(24) The spaces $C[a, b]$, l_p (with $1 \leq p < \infty$), and l_∞ are complete (with proofs).
(25) Completion of a metric space.
(26) Contraction mappings.
(27) Fixed point theorem.
(28) Applications of the Fixed point theorem.
(29) Topological spaces, axioms T1, T2, T3.

(30) Closed sets, closure, limit points, continuous maps, homeomorphism.
(31) Convergent sequences and limit points in general topological spaces.
(32) Hausdorff spaces, T_1-axiom.
(33) Quotient topology and its universal property.
(34) Examples of quotient spaces: the Möbius band, torus, Klein bottle and real projective plane.
(35) Compact spaces.
(36) Connected spaces.
(37) Path-connected spaces.
(38) Existence of solutions of differential equations (an application of the Contraction Mapping Theorem).