MTH 4104 Example Sheet II Shu SASAKI

II-1. Let R be the relation on the set of positive integers defined as follows: xRy if and only if there is no non-zero integer multiple of π between x and y. Prove that R is an equivalence relation (you may assume that π is irrational). Write down the equivalence class of 24 in R.

II-2. If r and s are real numbers, we define $\max(r,s)$ to be the greater one of r and s. For example, $\max(5,3) = 5$. Let R be the relation on $\mathbb{R}^2 = \{p = (x(p), y(p)) \in \mathbb{R}^2\}$ defined as follows: pRq if and only if $\max(|x(p)|, |y(p)|) = \max(|x(q)|, |y(q)|)$. Assuming that this defines an equivalence relation, state the partition of $\mathbb{R}^2$ defined by R.

II-3. Let S be any set set and T be any subset of S. Then $\{T, S - T\}$ is a partition of S. This assertion is missing a necessary assumption. Set it right by including the necessary assumption and then prove it.

II-4. Prove that, for any positive integer n, if X and Y are congruence mod n classes, then the set $\{x + y \mid x \in X, y \in Y\}$ defines a mod n congruence class. Show that this equivalence class is indeed $X + Y$ in $\mathbb{Z}_n$.

II-5. Give an example of a positive integer n and two mod n congruence classes X and Y such that the set $\{xy \mid x \in X, y \in Y\}$ does not define a mod n congruence class– in particular it does not equal XY in $\mathbb{Z}_n$

II-6. Write out addition and multiplication table for $\mathbb{F}_5$.

II-7. Find all solutions in x for $x^2 = 3x$ in $\mathbb{Z}_{10}$.

II-8. Let n be a positive integer,. Prove that the sum of all elements of $\mathbb{Z}_n$ equals $[0]_n$ if and only if n is odd.

II-9. Write out the multiplication table for $\mathbb{Z}_6$. How many times does $[0]_6$ appear in each row of the table? Based on your answer, write down a formula for how many times $[0]_n$ appear in the $[a]_n$ row of the multiplication table for $\mathbb{Z}_n$ for any integer n and a positive integer n.

II-10. Which element $[x]_{17}$ of $\mathbb{Z}_{17}$ satisfy the equation $[9]_{17}[x]_{17} + [1]_{17} = [11]_{17}[9]_{17}^{-1}$?

II-11. How many of the elements of $\mathbb{Z}_{19}$ have multiplicative inverses? What about $\mathbb{Z}_{20}$? What about $\mathbb{Z}_{66}$?