MTH6157 Survival Models

Practice Questions on Statistical Tests – Solutions

Question 1

(a) The null hypothesis is that the standard table values accurately represent the mortality experience for policyholders in their fifties.

Age	Exposed to Risk	Observed deaths	Standard table rate	Expected deaths	z_x	z_x^2
50	4100	45	0.011	45.1	-0.015	0.00022
51	4555	54	0.012	54.7	-0.089	0.00797
52	4505	61	0.013	58.6	0.318	0.10124
53	3900	59	0.015	58.5	0.065	0.00427
54	3995	65	0.018	71.9	-0.815	0.66400
55	4250	71	0.022	93.5	-2.327	5.41444
56	3060	80	0.027	82.6	-0.288	0.08308
57	2465	90	0.035	86.3	0.401	0.16083
58	2015	99	0.046	92.7	0.655	0.42956
59	1680	105	0.058	97.4	0.766	0.58655
Total						7.45217

the sum of z_x^2 values is 7.45

we compare this to the χ^2 critical value on 10 degrees of freedom because we are testing over 10 years of age

$\chi^2_{0.95;\,10} = 18.31 > 7.45$

therefore we do not reject the null hypothesis

the standard table values accurately represent the mortality experience for policyholders in their fifties

(b) This would not detect:

- a large deviation offset by many small deviations (outliers)
- small overall positive or negative bias
- a run or clump of ages with a positive or negative bias

(c) Use:

- standardised deviations test
- signs test (or the cumulative deviations test)
- grouping of signs test (also called Steven's test)

Question 2

(a) the mortality policy is a concern because:

- would expect to use different mortality tables for term assurance and annuity calculations
- partly because the financial risk relating to mortality estimation is the opposite way around for these two classes of business
- a standard table that is 'many years' old will not reflect more recent mortality trends
- as mortality rates tend to fall over time this is particularly concerning for annuity rate calculations
- use of incorrect tables over multiple products opens the company to the risk of adverse selection

(b) from looking at the z_x values:

- there are more positive than negative values
- 3 of 16 values are > 2 (whereas for N(0,1) samples we would expect less than 5% to be >2)
- we would hope that term assurance policyholder mortality would be lower than general population mortality (because of company underwriting practice)
- positive z_x values would be evidence of term assurance mortality experience being higher than the standard table not lower

(c) the overall test of goodness of fit is the chi-squared test

the null hypothesis is that the standard table remains a good fit for mortality experience amongst term assurance policyholders [Note – need to stress the investigation is amongst term assurance policyholders whilst the use of the table has been across term assurance and annuities – therefore the conclusions can only apply to term assurance business here.]

age x	z_x	z_x^2
35	0.832	0.692
36	2.343	5.490
37	-0.599	0.359
38	-0.458	0.210
39	-0.791	0.626
40	2.228	4.964
41	-0.783	0.613
42	1.334	1.780
43	0.230	0.053
44	0.595	0.354
45	2.465	6.076
46	-1.529	2.338
47	0.436	0.190
48	-0.663	0.440
49	0.287	0.082
50	1.387	1.924
sum		26.190

we have 16 ages of data so we compare the statistic above with the upper 5% critical value of the chi-squared distribution on 16 degrees of freedom

$\sum z_x^2 = 26.190 < 26.296 = \chi^2_{0.95;16}$

therefore we do not reject the null hypothesis at the 95% significance level

based on this chi-squared test the standard table remains a adequate fit for the term assurance mortality experience

however we note that the test statistic is very close to the critical value at 95% and so acceptance / rejection of the null hypothesis is very marginal

(d) conclusions we can draw:

- from (c) above we conclude the overall fit of the existing standard table to term assurance mortality experience is close enough not to reject the null hypothesis using the chi-squared test

- however the test statistic is very close to the critical value suggesting further investigation is warranted
- there are differences between the experience and the table that are not detected by the chi-squared test
- in particular the number of z_x values >2 could be investigated using the standard deviations test
- at these ages the rate of mortality is typically very low so we would not expect large deviations
- also, the greater number of positive z_x values could be investigated by the signs test
- if this standard table has been tested against experience before (given it has been in use for many years) we could compare our chi-squared test with previous ones to see whether there is evidence of deterioration in goodness-of-fit over the years (evidence of time selection perhaps)