MTH5126 - Statistics for Insurance

Worksheet 1

Question 1.

Derive the formula for the MGF of the standard normal distribution.

Hint: Complete the square in the exponent.

Question 2.

A random variable X follows a gamma distribution with parameters α and λ.

(i) Derive the moment generating function (MGF) of X.

(ii) Derive the coefficient of skewness of X.

Question 3. MLE, Method of moments, Method of percentiles

A random sample $x_1, x_2, \ldots, x_{20}$ is taken from a distribution having the density function:

$$f(x) = \frac{k}{5} x^{-\frac{4}{5}} e^{-kx^{\frac{1}{5}}}, x > 0$$

For this sample:

$$\sum_{i=1}^{20} x_i = 247,360$$

$$\sum_{i=1}^{20} x_i^{\frac{1}{5}} = 102.778$$

and the median is 10,000.

Determine the:
(a) maximum likelihood estimate of k.
(b) method of moments estimate of k.
(c) method of percentiles estimate of k.

Hint: For part b, compare the density function in the question with the PDFs of the distributions you saw in lectures.

Question 4. R

In-built functions can be found in R for the distributions in the table below. The table below shows the R functions you need for a series of calculation.

(i) Complete the table below.

Hint: Feel free to search online. Or type and run `?dgamma` and Help notes will appear. Similarly for the other functions.

Calculation required	*X~Exp(λ)*	*X~Gamma(α, λ)*	*X~logN(μ, σ^2)*	*X~Weibull(c,γ)*
f(x)	`dexp(x, λ)`			
F(q)	`pexp(q, λ)`			
Find *q* for *P(X< q) = p*	`qexp(p, λ)`			
Simulate *n* random variates	`rexp(n, λ)`			

(ii) Use R to find the median for the following distribution: *Weibull*(0.1099, 0.2).

Question 5. R

There are no in-built functions in R for the *Pareto(α, λ)* distribution. So we have to define the functions `dpareto, ppareto, qpareto` and `rpareto` from first principles. This has been done for `dpareto` as follows.

```
dpareto <- function(x,a,lambda){
  a*lambda^(a)/( (lambda+x)^(a+1) )
}
```

(i) Write down the R code for `ppareto, qpareto` and `rpareto`, and paste your coding into your answer.

Hint: `runif` generates random numbers which are between 0 and 1. Use this when you define `rpareto`.

(ii) Simulate, using `rpareto`, 1,000 values from a Pareto distribution with parameters $\alpha = 3$, $\lambda = 1$, assigning the simulation to a variable called `Pareto_vector` and calculate the mean and variance of the simulated values.

Hint: Use the `mean` and `var` functions in R.