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MTH5126 Statistics for Insurance

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Week 10

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Run-off triangles

Introduction

Run-off triangles or delay triangles are an important topic in the practical work of actuaries working in general insurance who make use of Excel spreadsheets and other software packages to forecast claim numbers and amounts.

We will look at four standard methods for projecting run-off triangles:

1. Basic chain ladder method
2. Inflation-adjusted chain ladder method
3. Average cost per claim method
4. Bornhuetter-Ferguson method

- For the average cost per claim and the Bornhuetter-Ferguson methods, there is no one way of applying the method that is universally accepted.
- If you go on to use this in the “real world” then you will see variations in everyday use.
- In an exam situation you should apply these methods sensibly to the data you are given.

Run-off triangles

Introduction

- Run-off triangles (delay triangles) usually arise in types of insurance (particularly non-life insurance) where it may take some time after a loss until the full extent of the claims which have to be paid is known. It is important that the claims are attributed to the year in which the policy was written.
- We analyse the claims in cohorts and it is important that each claim is allocated to the correct cohort.
- For example, claims can be grouped by the year in which the policy was originally written, or by the year in which the accident occurred, or in a number of other ways.
- The insurance company needs to know how much it is liable to pay in claims so that it can calculate how much surplus it has made.
- However, it may be many years before it knows the exact claims totals. There are many causes for the delays in the claim totals being finalised.

Run-off triangles

Introduction

- The delay may occur before notification of the claim and/or between notification and final settlement.
- Although the insurance company does not know the exact figure for total claims each year, it must try to estimate that figure with as much confidence and accuracy as possible.

- The main question we will attempt to answer is this:

How much needs to be set aside now, as a reserve, to meet future payments on claims that have arisen during some recent period?

Types of reserves

General insurers need to be able to estimate the *ultimate* cost of claims for several reasons.

- One of these is that they need to know the full cost of paying claims in order to set future premium rates.
- They also need to set up reserves in their accounts to make sure that they have sufficient assets to cover their liabilities.
- The usual steps involved in settling a general insurance claim are:

Claim event occurs → Claim reported → Claim payment made → Claim file closed

- After a claim event has occurred (for example a policyholder has been involved in a motor accident or has been burgled), the policyholder will report the incident to the insurer.
- In due course the insurer will make any payments required (for example paying for repairs to a vehicle, compensation to an injured person, or the cost of replacing stolen belongings).
- There may be several payments under a single claim.

Types of reserves

- When the insurer considers that no further payments will be required for the claim, then the claim file will be closed.
- A general insurer will need to set up reserves to cover its liabilities for future payments in respect of accidents that have already occurred. These reserves will relate to claims at different stages in the settlement process.
- In particular reserves will be required for *IBNR* claims and *outstanding reported claims.*

IBNR

An IBNR reserve is required in respect of claims that have been *incurred but not reported*.

This means the claim event has occurred but the claim has not yet been reported to the insurer.

Outstanding reported claims

An *outstanding reported* claims reserve is required in respect of claims that have been reported, but have not yet been closed.

Presentation of claims data

There are several ways of presenting claims data, which emphasise different aspects of the data.

- Here they will be presented as a triangle, which is the most commonly used method.
- The year in which the incident happened and the insurer was on risk is called the accident year.
- The number of years until a payment is made is called the delay, or development period.
- The claims data are divided up by the accident year and the development year.
- The following table is an example of claims data referenced by accident year and development year.
- In some types of insurance it might be relevant to look at development of claims by month or quarter, but the principles are unchanged.
- The data can be presented either cumulatively, or on an individual year basis.

Presentation of claims data

Example

Accident Year	Development Year				
	0	**1**	**2**	**3**	**4**
2008	786	1,410	2,216	2,440	2,519
2009	904	1,575	2,515	2,796	
2010	995	1,814	2,880		
2011	1,220	2,142			
2012	1,182				

This table shows cumulative claim payments

- ✓ Each row in the triangle represents an *origin* year which defines a *cohort* of claims.
- ✓ This example uses an *accident year* cohort.
- ✓ The columns represent *development* years, which show how the cohort of claims relating to a particular origin year "develop" over time.
- ✓ Column 0 represents the year in which the accident occurred. Column 1 represents the year after the accident occurred, etc.

Presentation of claims data

Example

- ✓ Each entry in the table can be defined by its *accident year* - **row** and its *development year* - **column**.
- ✓ For example the figure of 2,216 is for Accident Year 2008, Development Year 2, which we will write as 2008/2 or $C_{2008,2}$.
- ✓ Note that this figure includes payments made in 2008, 2009 and 2010 since this table is cumulative.
- ✓ The figures given are cumulative and represent total amounts paid by the end of each development year.
- ✓ They have been compiled after the end of the 2012 accident year.
- ✓ For the 2011 accident year, only payments with delay 0 and delay 1 have been reported and so on.

Question:

Use the delay triangle given to determine:

(i)The total amount of claims paid in 2012 in respect of accidents that occurred in 2010, and

(ii)The total amount of claims paid during 2012.

Presentation of claims data

Example

Solution:

Because the triangle shows cumulative amounts, we need to subtract neighboring columns.

(i) 2,880 - 1,814 = 1,066

(ii) 1,182 + (2,142 - 1,220) + (2,880 - 1,814) + (2,796 - 2,515) + (2,519 - 2,440) = 3,530

Helpful notes:

- Particular calendar years are represented by the *diagonals* in the triangle. For example the long diagonal (1182, 2142, 2880, 2796, 2519) includes all payments made during the most recent calendar year shown (2012 in this case) as well as past calendar years.
- Also, the upper left corner represents "known" past payments.
- Similarly the lower right corner represents "unknown" future payments.

Estimating future claims

Example

- Our task is to look at methods of estimating these unknown figures to complete the lower right triangle.
- For each accident year, the difference between the figure in the extreme right hand column and the total amount paid so far will give us the estimate of the amount we need to hold currently to meet future liabilities arising.
- So the task is to decide the amounts yet to be paid in respect of the given accident years.
- This can be done for 2012 by looking at previous accident years.
- If the cumulative payments increase in a similar way, it is possible to say that they are likely to be about 3,788 in 4 year's time.
- This figure is obtained by assuming that the 2012 accident year is similar to the 2008 accident year in the pattern of making payments and estimating cumulative payments at the end of Development Year 4 by:

$$1,182 \times \frac{2,519}{786} = 3,788$$

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Projections using development factors

- The basic assumption made in estimating outstanding claims concerns the run-off pattern.
- The simplest assumption is that payments will emerge in a similar way in each accident year.
- The proportionate increases in the known cumulative payments from one development year to the next can then be used to calculate the expected cumulative payments for future development years.
- However, as our next example shows, there are a number of choices as to which such ratio should be used to project future claims.
- Note: the ratios that are used to project future claims are known as **development factors** or **link ratios**.

Projections using development factors

Example

Accident Year	Development Year								
	0		**1**		**2**		**3**		**4**
2008	786	*1.794*	1,410	*1.572*	2,216	*1.101*	2,440	*1.032*	2,519
2009	904	*1.742*	1,575	*1.597*	2,515	*1.112*	2,796		
2010	995	*1.823*	1,814	*1.588*	2,880				
2011	1,220	*1.756*	2,142						
2012	1,182								

Proportionate increases in cumulative claim payments

- ✓ For each accident year from 2008 to 2011 there is a different ratio for the increase in cumulative payments from Development Year 0 to Development Year 1.
- ✓ It is not clear which is the “correct” one to use when projecting forward for Accident Year 2012.
- ✓ For a conservative estimate of cumulative payments, it might be best to take the largest ratio, *ie* 1.823.

Projections using development factors

Example

Question: Project the known figure for Accident Year 2012 across to Development Year 4 using:

(i) The largest ratio for each development year

(ii) The smallest ratio for each year.

(iii) Comment on the results.

Solution:

(i) Using the largest ratio for each development year we get:

$$1,182 \times 1.823 \times 1.597 \times 1.112 \times 1.032 = 3,949$$

(ii) Using the smallest ratio in each case we get:

$$1,182 \times 1.742 \times 1.572 \times 1.101 \times 1.032 = 3,678$$

(iii) The results are quite different.

The estimate for the outstanding claims reserve will be very sensitive to the method used to calculate the development factors.

Projections using development factors

Arithmetic average

- It should be obvious that some sort of average of the ratios would be more appropriate.

  We could just use a simple arithmetic average:

$$\frac{1.794 + 1.742 + 1.823 + 1.756}{4} = 1.779$$

- The disadvantage of this is that it does not take into account that the years in which more claims occur provide more information.
- So, the greater the amount of claims, the more confidence we can have in the ratio.

Projections using development factors

Weighted average

- This suggests using a weighted average and the usual choice of weights are the cumulative claims values.

Accident Year	Ratio	Weight
2008	1.794	786
2009	1.742	904
2010	1.823	995
2011	1.756	1,220

$$\frac{1.794 \times 786 + 1.742 \times 904 + 1.823 \times 995 + 1.756 \times 1,220}{786 + 904 + 995 + 1,220} = 1.777$$

- This method of estimating the ratios which describe the run-off pattern is called **the chain ladder method.**

A statistical model for run-off triangles

The general form of a run-off triangle can be expressed as follows:

Accident Year	**Development Year**							
	0	**1**	$\dots$	**j**	$\dots$	$\dots$	$\dots$	**n**
0	$C_{0,0}$	$C_{0,1}$	$\dots$	$C_{0,j}$	$\dots$	$\dots$	$\dots$	$C_{0,n}$
1	$C_{1,0}$	$C_{1,1}$	$\dots$	$C_{1,j}$	$\dots$	$\dots$	$C_{1,n-1}$	
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$		
i	$C_{i,0}$	$C_{i,1}$	$\dots$	$C_{i,j}$	$C_{i,n-i}$			
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$				
$\dots$	$\dots$	$\dots$	$\dots$					
$\dots$	$\dots$	$C_{n-1,1}$						

A statistical model for run-off triangles

Notation

Each entry, C_{ij}, in the run-off triangle represents the incremental claims (as opposed to cumulative claims) and can be expressed in general terms.

$$C_{ij} = r_j \times s_i \times x_{i+j} + e_{ij}$$

where:

$\mathbf{r_j}$ is the development factor for year j, representing the proportion of claim payments in Development year j. Each r_j is independent of the Origin Year i.

$\mathbf{s_i}$ is a parameter varying by Origin Year, i, representing the exposure, for example the number of claims incurred in the Origin Year i.

$\mathbf{x_{i+j}}$ is a parameter varying by calendar year, for example representing inflation.

$\mathbf{e_{ij}}$ is an error term.

A statistical model for run-off triangles

Notes

- Note that some of the terminology here is being used in a slightly different context.
- The development factors r_j in this general statistical model are defined differently to the development factors that we met before.
- Those we met before were used to project forward *cumulative* claims data.
- These r_j development factors in this statistical model are, however, used to model *incremental* data.
- They are defined above as the proportion of claims from a particular accident year that are paid in the j^{th} development year.
- As such they are a set of factors that add up to 1.

The chain ladder method

Example:

Recall that the ratio in Accident Year 2008 was calculated as follows:

$$1.794 = \frac{1,410}{786}$$

The ratios for the other accident years were calculated in a similar way. The numerator of the equation we met in our weighted average section can therefore be written as:

$$\frac{1,410}{786} \times 786 + \frac{1,575}{904} \times 904 + \frac{1,814}{995} \times 995 + \frac{2,142}{1,220} \times 1,220$$

$$= 1,410 + 1,575 + 1,814 + 2,142$$

Thus the development factor can be calculated using the cumulative claims in Development Years 0 and 1.

$$\frac{1,410 + 1,575 + 1,814 + 2,142}{786 + 904 + 995 + 1,220}$$

The chain ladder method

- In other words the development factor is the sum of the figures in Column 1 divide by the sum of the *Corresponding figures* from Column 0.
- The name given to this method presumably arises from the ladder-like operations which are chained over the development years.
- The development factors for the chain ladder technique can be found for each development year by adding the appropriate number of terms. We now illustrate this.

Accident Year	Development Year				
	0	**1**	**2**	**3**	**4**
2008	786	1,410	2,216	2,440	2,519
2009	904	1,575	2,515	2,796	
2010	995	1,814	2,880		
2011	1,220	2,142			
2012	1,182				

The chain ladder method

$$\text{Development factor } 0 \rightarrow 1 = \frac{6{,}941}{3{,}905} = 1.777$$

$$\text{Development factor } 1 \rightarrow 2 = \frac{7{,}611}{4{,}799} = 1.586$$

$$\text{Development factor } 2 \rightarrow 3 = \frac{5{,}236}{4{,}731} = 1.107$$

$$\text{Development factor } 3 \rightarrow 4 = \frac{2{,}519}{2{,}440} = 1.032$$

- Note that development factors will normally be 1-point-something.
- With development factors calculated for each development year it is now possible to project forward each accident year.
- For Accident Year 2012, the projections of cumulative claims are:

$$\begin{aligned}
&1,182 \times 1.777 && = 2,100 \\
&1,182 \times 1.777 \times 1.586 && = 3,331 \\
&1,182 \times 1.777 \times 1.586 \times 1.107 && = 3,688 \\
&1,182 \times 1.777 \times 1.586 \times 1.107 \times 1.032 && = 3,806
\end{aligned}$$

The chain ladder method

- For Accident Year 2011, start from 2,142 in Development Year 1 and use only the last 3 link ratios.

Accident Year	Development Year				
	0	1	2	3	4
2008					
2009					2,885
2010				3,188	3,290
2011			3,397	3,761	3,881
2012		2,100	3,331	3,688	3,806

- Note that no projection can be done for the first accident year because it is not possible to project beyond the highest development year.

The chain ladder method

- The reserve that needs to be held at the end of 2012 is the sum over all accident years for which a projection has been made of the difference between the cumulative payment at the end of Development Year 4 and the last known entry in the development triangle for that accident year.
- So, the reserve at the end of 2012 is:

$$(2,855-2,796)+(3,290-2,880)+(3,881-2,142)+(3,806-1,182) = 4,862$$

- Note that no discount rate has been applied to the payments in different years.

Homework

- **Please try the exercise above using Excel spreadsheet and see if you can reproduce the same answers.**

The inflation-adjusted chain ladder method

Accident Year	Development Year				
	0	1	2	3	4
2008	786	1,410	2,216	2,440	2,519
2009	904	1,575	2,515	2,796	
2010	995	1,814	2,880		
2011	1,220	2,142			
2012	1,182				

This table shows cumulative claim payments

- We will use the above table to illustrate the inflation-adjusted chain ladder method.

The inflation-adjusted chain ladder method

Dealing with past inflation

Claims inflation will affect the payments in the run-off triangle by calendar year of payment. In the model we now consider, it will be assumed that claims inflation is at the same annual rate for all claims within a particular calendar year of payment.

- Each calendar year of payment corresponds to a diagonal in the triangle.
- You can see this in the table on the previous slide.
- When adjusting for inflation, it is the payments in each calendar year which need to be considered, rather than cumulative totals.
- The first step is to calculate incremental payments from the cumulative totals, by differencing along each row.

Accident Year	**Development Year**				
	0	**1**	**2**	**3**	**4**
2008	786	624	806	224	79
2009	904	671	940	281	
2010	995	819	1,066		
2011	1,220	922			
2012	1,182				

This table shows incremental (i.e. non-cumulative) claim payments

The inflation-adjusted chain ladder method

Dealing with past inflation

- Suppose now that annual claim payments inflation rates over the 12 months up to the middle of the given year are as follows:

2009	5.1%
2010	6.4%
2011	7.3%
2012	5.4 %

- For simplicity, it is also assumed that payments are made in the middle of each calendar year.
- An index can now be calculated in order to convert all payments to mid-2012 prices.

The inflation-adjusted chain ladder method

Dealing with past inflation

Question:

What would the index be for the years 2008-2012 if the base value is 100 for 2008?

Solution:

2008	100
2009	105.1
2010	111.8
2011	120.0
2012	126.5

To inflation- adjust a payment (from 2008) we can either perform the following calculation:

$$786 \times 1.051 \times 1.064 \times 1.073 \times 1.054 = 994$$

or we can use the index.

$$786 \times \frac{126.5}{100} = 994$$

The inflation-adjusted chain ladder method

Dealing with past inflation

Question:

Adjust the payment of 1,220 to mid-2012 prices.

Solution:

$$1,220 \times 1.054 = 1,286$$

or

$$1,220 \times \frac{126.5}{120} = 1,286$$

The inflation-adjusted chain ladder method

Dealing with past inflation

- The payments can now be adjusted using the inflation rates.

Accident Year	Development Year				
	0	**1**	**2**	**3**	**4**
2008	994	751	912	236	79
2009	1,088	759	991	281	
2010	1,125	863	1,066		
2011	1,286	922			
2012	1,182				

This table shows incremental (i.e. non-cumulative) claim payments at mid-2012 prices

The inflation-adjusted chain ladder method

Dealing with past inflation

- Now it is straightforward to form a table of inflation-adjusted cumulative payments to which the chain ladder technique can be applied.

Accident Year	Development Year				
	0	**1**	**2**	**3**	**4**
2008	994	1,745	2,657	2,893	2,972
2009	1,088	1,847	2,838	3,119	
2010	1,125	1,988	3,054		
2011	1,286	2,208			
2012	1,182				

This table shows cumulative claim payments at mid-2012 prices

- This gives us development factors in successive years of: 1.7334, 1.5321, 1.0941, 1.0273.

The inflation-adjusted chain ladder method

Dealing with past inflation

- We can use these development factors on the cumulative figures to now complete the table of cumulative payments at mid-2012 prices.

Accident Year	Development Year				
	0	1	2	3	4
2008					
2009					3,204
2010				3,341	3,432
2011			3,383	3,701	3,802
2012		2,049	3,139	3,434	3,528

This table shows projected cumulative claim payments at mid-2012 prices

- In practice you do not need to include column 0 or row 2008 in your answer.

The inflation-adjusted chain ladder method

Dealing with future inflation

- The predictions of cumulative payments do not, however, take account of future inflation.
- In order to forecast the actual payments, an assumed rate of future inflation will be needed.
- Again, it is necessary to convert to non-cumulative data rather than the cumulative totals before adjusting these for future inflation in a similar way to that used when dealing when past inflation.
- First we need to construct the table of incremental claims payments at mid-2012 prices. We do this by differencing the cumulative table.

Accident Year	**Development Year**			
	1	**2**	**3**	**4**
2009				85
2010			287	91
2011		1,175	318	101
2012	867	1,090	295	94

This table shows incremental claim payments at mid-2012 prices

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The inflation-adjusted chain ladder method

Dealing with future inflation

- Let's assume and apply a 10% future inflation rate.

Accident Year	Development Year			
	1	2	3	4
2009				94
2010			316	110
2011		1,293	385	134
2012	954	1,319	393	138

This table shows incremental claim payments.

The inflation-adjusted chain ladder method

Dealing with future inflation

- We can now accumulate using the original figures. (See table on slide 26).

Accident Year	Development Year				
	0	1	2	3	4
2008	786	1,410	2,216	2,440	2,519
2009	904	1,575	2,515	2,796	2,890
2010	995	1,814	2,880	3,196	3,306
2011	1,220	2,142	3,435	3,820	3,954
2012	1,182	2,136	3,455	3,848	3,986

This table shows cumulative claim payments.

The inflation-adjusted chain ladder method

Outstanding claims reserve

Question:

What is the outstanding claims reserve for the previous table?

Solution:

$$\text{Outstanding claims reserve}$$
$$= (3,986 - 1,182) + (3,954 - 2,142) + (3,306 - 2,880) + (2,890 - 2,796)$$
$$= 5,136$$

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