MTH5126 - Statistics for Insurance

Academic Year: 2022-23 **Semester: B**

Worksheet 9

Q1. Ruin Theory

An insurer calculates the annual premiums for fire insurance of flats by increasing the risk premium by 30% and adding a £30 loading.

The claim frequency is 3% and individual claim amounts can be assumed to be:
£2,000 with probability 0.9
£15,000 with probability 0.1

Calculate the insurer's adjustment coefficient for these policies, to 2 significant figures.

Q2. Ruin Theory

Show that the adjustment coefficient for a compound Poisson claims process satisfies the inequality:

$$R < \frac{2[\frac{c}{\lambda} - E(X)]}{E(X^2)}$$

and define what each of the symbols represents.

Q3. Ruin Theory

Claims on a portfolio of insurance policies arrive as a Poisson process with parameter λ, claim amounts having a Normal distribution with mean μ and variance σ^2, and there is a loading θ on premiums. The insurance company has an initial surplus of U.

(i) Explain carefully the meaning of $\Psi(U)$, $\Psi(U, t)$ and $\Psi(U,1)$.
(ii) State four factors that affect the size of $\Psi(U, t)$, for a given t.
(iii) Explain, for each factor, what happens to $\Psi(U, t)$ when the factor increases.

Sarah, the insurance company's actuary, prefers to consider the probability of ruin in discrete rather than continuous time.

(iv) Explain an advantage and disadvantage of Sarah's approach.

Q4. Excel-based

Please use the Excel template provided.

An insurance company has a portfolio of policies. Consider an aggregate claims process, $S(t)$, where:

- $S(t) = X_1 + X_2 + \dots + X_{N(t)}$
- $N(t)$ = the number of claims generated by the portfolio in the time interval $[0, t]$
- X_i = the amount of the i^{th} claim.

You have been given a single realisation of $N(5)$ and the values of the first three claim amounts (see Worksheet 'Data' in the template), which follow an exponential distribution with mean 1.

(i) Calculate the value of the aggregate claims process at time $t = 5$.

Assume that c, the rate of premium income per unit time, is 1.1 and U, the initial surplus, is 0.5.

(ii) Calculate the value of the surplus process, $U(t)$, at time $t = 5$.

$N(t)$ is a Poisson process with $\lambda = 1$, therefore the interval of time between claims is exponentially distributed with a mean of 1. You have been given realisations of the time intervals t_i between the first three claims, where t_1 is the time of the first claim and t_i (for $i > 1$) is the time between claim i and claim $i - 1$.

(iii) Plot a chart showing the surplus process from time $t = 0$ to time $t = 5$.

(iv) Calculate the minimum value of U, to three decimal places, that would avoid ruin before time $t = 5$ in the process in part (iii).