

MTH5126 - Statistics for Insurance

Academic Year: 2022-23

Semester: B

Worksheet 4

Q1.

Claims on a group of policies of a certain type arise as a Poisson process with parameter λ_1 .

Claims on a second, independent, group of policies arise as a Poisson process with parameter λ_2 .

The aggregate claim amounts on the respective groups are denoted S_1 and S_2 . Using moment generating functions, show that S (the sum of S_1 and S_2) has a compound Poisson distribution and hence derive the Poisson parameter for S .

Q2.

Consider aggregate claims over a period of 1 year, S , on a portfolio of general insurance policies:

$$S = X_1 + X_2 + \cdots + X_N$$

The number of claims each year, N , has a Poisson distribution with mean 12. X_1, X_2, \dots are assumed to be random variables, independent of each other and independent of N , with the following distribution:

$$f(x) = 0.01e^{-0.01x}, \quad 0 < x < \text{£}200$$

$$P(X = \text{£}200) = e^{-2}$$

The insurer effects excess of loss reinsurance with a retention limit of £100. Annual aggregate claims paid by the reinsurer are denoted by S_R . Calculate $E(S_R)$.

Q3. R

In an insurance company's portfolio, individual claim sizes, in £, follow an exponential distribution with parameter 0.0001.

- (i) Run the following code: `set.seed(123)` and then use R to simulate 10,000 claims and plot a histogram of the simulated data. Paste your R code and chart into your answer.
- (ii) Calculate the mean and variance of the simulated claims in part (i).

The insurer decides to take out an individual excess of loss reinsurance arrangement with a retention level of £20,000.

- (iii) Calculate the mean and the variance of the claims paid, under this arrangement, by
 - (a) the insurer
 - (b) the reinsurer.

The insurer wishes to determine an appropriate retention limit and has asked an analyst to investigate the effect of different retention limits on the mean and variance of claims.

- (iv) Calculate the mean and variance of the claims paid by:
 - (a) the insurer
 - (b) the reinsurer

under each of the following six retention levels:
£5,000, £10,000, £20,000, £30,000, £40,000 and £50,000.

- (v) Plot your results from part (iv) on four separate charts to show how the mean and variance of the claims paid by the insurer and reinsurer vary with the retention level.