Queen Mary
University of London

Late-Summer Examination period 2020

MTH6157: Survival Models

You should attempt ALL questions. Marks available are shown next to the questions.

In completing this assessment, you may use books, notes, and the internet. You may use calculators and computers, but you should show your working for any calculations you do. You must not seek or obtain help from anyone else.

At the start of your work, please **copy out and sign** the following declaration:

> I declare that my submission is entirely my own, and I have not sought or obtained help from anyone else.

All work should be **handwritten**, and should **include your student number**.

You have **24 hours** in which to complete and submit this assessment. When you have finished your work:

- scan your work, convert it to a **single PDF file** and upload this using the upload tool on the QMplus page for the module;
- e-mail a copy to maths@qmul.ac.uk with your student number and the module code in the subject line;
- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

You are not expected to spend a long time working on this assessment. We expect you to spend about **2 hours** to complete the assessment, plus the time taken to scan and upload your work. Please try to upload your work well before the end of the assessment period, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final**.

IFoA exemptions
This module counts towards IFoA actuarial exemptions. For your submission to be eligible for IFoA exemptions, you must submit within the first **3 hours** of the assessment period. You may then submit a second version later in the assessment period if you wish, which will count only towards your degree. There are two separate upload tools on the QMplus page to enable you to submit a second version of your work.

Examiners: C. Sutton, M. Poplavskyi

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Question 1 [18 marks]. Guests at the SeaView Hotel get breakfast by queuing and then collecting their food and drink from a buffet laid out on a large table. The hotel manager is concerned that guests have to wait too long in queue and decides to conduct a study using a Kaplan-Meier estimate. Breakfast is served from 7:00 until 10:00 am and the manager measures the queuing times in minutes for the first 30 guests arriving after 7:30 one morning. These are:

2, 5, 6, 2, 6*, 2, 8, 2, 1, 4, 6*, 3, 2, 7, 6, 7, 4, 3, 5, 4*, 3, 2, 2*, 3, 6, 3, 1, 8, 1, 4

where * represents a guest who left the queue before getting their breakfast.

(a) Calculate the Kaplan Meier estimate of the survival function for this queue. [**12**]

(b) The manager would like to claim in advertisements that 90% of guests queue for fewer than X minutes. Based on this study what should the value of X be? [**2**]

(c) Give reasons why this study might not give a reliable result for X. [**4**]

Question 2 [21 marks]. A study seeks to compare mortality for elderly people with dementia living in Glasgow and Paris. The results are compared using a Cox's Proportional Hazard Model and the model fitted is

$$h_i(t) = h_0(t)exp(\beta\ z^T)$$

where
$h_i(t)$ is the hazard at time t weeks
$h_0(t)$ is the baseline hazard.
$z = (z_1, z_2, z_3)$ is a vector of covariates where z_1 is 1 for people living in Glasgow and 0 for those in Paris; z_2 is the number of weeks since the person was last seen by their doctor; and z_3 is 1 if the person lives in a care home and 0 if they live in their private home.
$\beta = (\beta_1, \beta_2, \beta_3)$ is a vector of parameters where $\beta_1 = -0.1$; $\beta_2 = 0.05$; $\beta_3 = 0.4$.

(a) State the group of lives to whom the baseline hazard applies. [**5**]

(b) For someone living in a care home in Glasgow who saw their doctor 4 weeks ago:
(i) Write down the hazard function in terms of the baseline hazard.
(ii) Find an expression for the survival function in terms of the baseline hazard. [**6**]

(c) The probability that a person living in a care home in Paris who was seen by their doctor two weeks ago is still alive 26 weeks later is 0.985. Calculate the probability that the person in (b) above will survive 26 weeks. [**10**]

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Question 3 [19 marks]. A new smart watch shows whether the wearer's heart rate over the past hour averaged more than 85 beats per minute (BPM) or not. The watch is used on 200 patients who are recovering from a heart attack to investigate the relationship between recent heart rate and mortality.

(a) Draw a multi-state model that could be used for this investigation, labelling and defining all transition intensities. [**7**]

(b) Write down an expression for the likelihood of the data in terms of transition intensities and waiting times, defining all the terms you use. [**4**]

(c) The investigation observes a total of 1406 hours of average heart rate less than 85 BPM, 1738 hours of average heart rate above 85 BPM, 68 transitions from low to high average heart rate, 97 transitions from high to low average heart rate and 18 deaths in total with 8 of these having an average heart rate below 85 BPM in the hour before death. Calculate the maximum likelihood estimator for the transition intensity from high heart rate to dead. [**5**]

(d) Why should the investigators be careful interpreting the results from this model? [**3**]

Question 4 [21 marks]. For a number of years, a life assurance company has produced its own mortality tables by graduating observed mortality rates with reference to a parametric formula based on Makeham's Law. A senior actuary is concerned that this method of graduation is no longer producing satisfactory results, particularly at older ages. They compare raw and graduated mortality rates at ages 85 and above, computing the following standardised deviations z_x at age x:

Age	85	86	87	88	89	90	91	92	93
z_x	0.4	0.3	-0.6	-1.5	-0.2	0.1	-0.5	0.2	1.2
Age	94	95	96	97	98	99	100	101	102
z_x	0.3	0.4	0.9	-0.2	0.4	-0.5	0.9	0.4	0.6

(a) What are the disadvantages of the graduation method being used? [**2**]

(b) Give three ways in which the graduation method could be improved. [**3**]

(c) Complete a test for the overall goodness-of-fit of the graduation. [**9**]

(d) Why does the test in (c) above give an incomplete analysis of the quality of this graduation? [**4**]

(e) What particular issues does the actuary need to be aware of for this age group? [**3**]

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Question 5 [21 marks]. A life assurance company uses a Poisson model to estimate mortality by age when calculating annuity values. The actuary is concerned that this approach does not take account of the adverse impact of smoking. The company holds data for each policyholder which includes date of birth, number of other life assurance policies owned, whether they are a smoker and whether the policy was bought direct online or through a sales agent.

(a) What types of selection is the actuary concerned about here? [**3**]

(b) List the three stages to building a Generalised Linear Model for mortality. [**6**]

(c) Specify fully a Generalised Linear Model for mortality which uses the data available to this life assurance company. [**9**]

(d) Describe how the actuary could use the model in (c) above to test the mortality rates used by the company given the concern about smokers. [**3**]

End of Paper.