

# Mortality Projections

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# Who needs to project future mortality rates?

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- life assurance actuaries
- pension actuaries
- governments
  - Social security systems
  - Health and Care

# 3 main approaches

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**Expectation**

**Extrapolation**

**Explanation**

# Projecting by Expectation

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# Projections based on Expectation

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Method historically favoured by government statistical agencies

Idea is to set an 'expectation' of future declines in mortality rates

2 main ways

- assume continuation of recent year rate of change
- convene group of experts to set a target for future reductions

# Setting targets with Reduction Factors

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Define  $R_{x,t}$  the proportion by which  $q_x$  expected to reduce by some future year  $t$

So if  $q_{x,0}$  is the mortality rate at age  $x$  now at  $q_{x,t}$  is the rate in future year  $t$  then

$$q_{x,t} = R_{x,t} q_{x,0}$$

Where  $R_{x,t}$  is of the form

$$R_{x,t} = a_x + (1 - a_x)(1 - f_{n,x})^{t/n} \text{ for } 0 < n < t$$

with

$a_x$  = the ultimate reduction factor, and

$f_{n,x}$  = the proportion of decline occurring in  $n < t$  years

This gives an exponential shape to the decline in  $q_x$  rather than a linear one

# Advantages & Disadvantages

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- easy to understand and implement



- historically these methods have **underestimated** mortality improvements
- change in smoking
- improvements in prevention and treatment of heart disease and strokes
- problems of equating “targets” with “forecasts”

# Possible developments

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Use of large sample size questionnaires on health and wellbeing

- possible that self-reported health status more valuable than expert targets
- care needed that sample reflects population health factors
- potential data science application



# Projecting by Extrapolation

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# Stochastic versus Deterministic approach

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Methods based on expectation are essentially deterministic with fixed parameters inserted

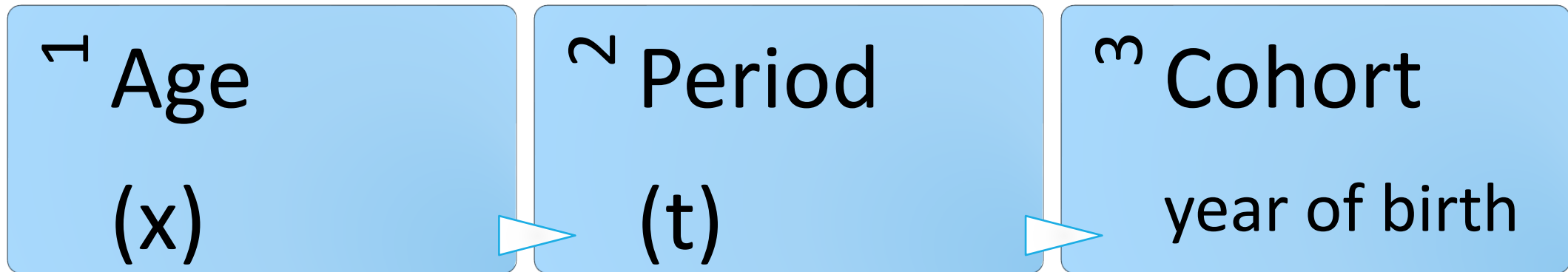
For most investigations today, stochastic approaches are preferred – models that depend on a number of factors

This underlies the extrapolation based forecasting methods

The idea is that given some rates of mortality based on some (recent) past time period, we attempt to estimate rates of mortality for some future time period on the basis of one or more factors and time  $t$ .

# Factors that apply to mortality forecasts

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*Note that Cohort is not independent of Age and Period*

Extrapolation methods create models of how mortality rates change over time with 1, 2 or 3 of these factors using past factor and mortality data to fit the model

# The Lee-Carter Model

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# The Lee-Carter model

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One of the most widely used models in forecasting future mortality rates

Two factor extrapolation model using Age + Period

Lee & Carter (1990s) wrote the original model in terms of **central rate of mortality** at some future time  $t$  denoted  $m_{x,t}$

- See week 2 notes for definition of  $m_x$  and relation to  $q_x$

$$\log m_{x,t} = a_x + b_x k_t + \varepsilon_{x,t}$$

# Lee-Carter model (continued)

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$$\log m_{x,t} = a_x + b_x k_t + \varepsilon_{x,t}$$

Where

$a_x$  describes the general shape of mortality at age  $x$

$k_t$  measures the general population change in mortality in period to time  $t$

$b_x$  is the rate of response to the  $k_t$  change for age  $x$

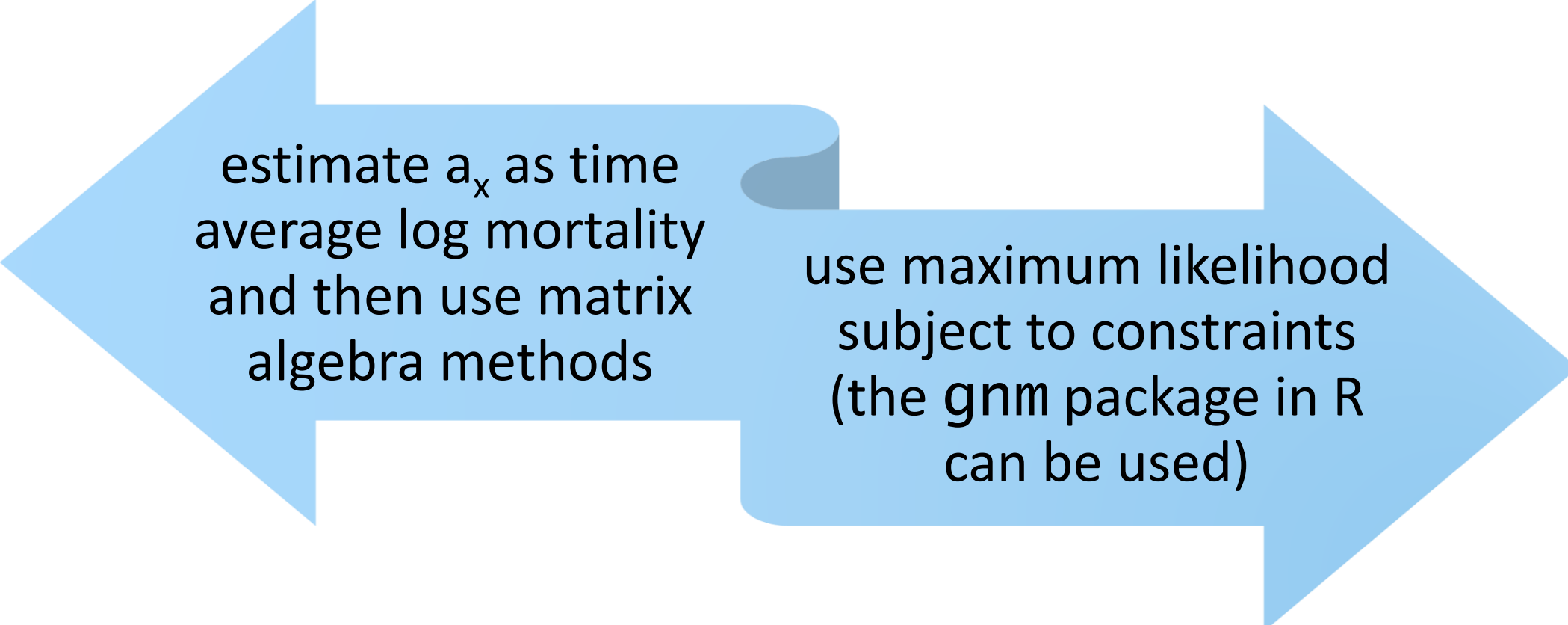
$\varepsilon_{x,t}$  are set of independent  $N(0, \sigma^2)$  variables where  $\sigma$  is to be estimated

to find  $a$ ,  $b$ ,  $k$ ,  $\varepsilon$  parameters some constraints are needed and usual ones are

$$\sum_x b_x = 1 \text{ and } \sum_t k_t = 0$$

# Estimation in the Lee-Carter model

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estimate  $a_x$  as time average log mortality and then use matrix algebra methods

use maximum likelihood subject to constraints (the `gnm` package in R can be used)

# Using the Lee Carter model for forecasts

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The model has three parameters, two depending on age  $x$  and one ( $k_t$ ) on time  
Forecasting mortality then involves forecasting  $k_t$  whilst keeping  $a$  and  $b$  constant

The natural way to do this is by **Time Series** methods

- Random walk models
- Moving average models
- Auto regressive models

You will learn more about these in module MTH6139 next Semester

The key advantage of Lee Carter is that it is straightforward to apply using Time Series techniques once parameters have been estimated



# Disadvantages of Lee Carter

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- ❑ forecasts lose the smoothness of an original graduation over time
- ❑ using random walk time series methods can lead to relationships between mortality at different ages which we know is not reflected in practice
- ❑ there is no Cohort term, only Age and Period
  - ❑ we know from other investigations that UK mortality does exhibit Cohort effects over time (e.g. related to smoking patterns, wartime, COVID-19)
- ❑ future estimates become heavily dependent on the  $a_x$  and  $b_x$  parameters which are generally estimated from past data which might become less reliable as a guide to the future

Some forecasters have adapted Lee Carter to a 3 factor model including Cohort – we will not cover these in detail in this module

# Splines and p-splines

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We introduced Splines last week as a method of graduation

- polynomials of a specified degree joined piecewise at knots

It is also possible to use Splines in forecasting future mortality

- define  $\log E(D_x)$  as a polynomial function
- the more knots we have the better the combined spline function will adhere to past mortality data but the less smooth the result will become
- p-splines is a method for optimising the number of splines and knots
- this can be done with the `MortalitySmooth` package in R

# Projecting by Explanation

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# Factors affecting mortality

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The expectation and extrapolation methods do not use scientific knowledge on the factors affecting mortality and on medical advances

E.g. developments in cancer detection and treatment

However:

- modelling medical risk factors is as complicated as modelling mortality
- the time series data is not always of great quality or length
- cause of death data is not always reliable worldwide

# Sources of error

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# Mortality forecasts are always wrong!

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Survival Model badly specified

Uncertain parameter estimates

Data set does not reflect population

Climate change

Random variation and data errors

# Further Reading on Mortality Projections

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Booth, H. and Tickle, L. (2008). *Mortality modelling and forecasting: a review of methods*, Australian Demographic and Social Research Institute Working Paper no. 3, Canberra, Australian National University.

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