Mortality Projections

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Who needs to project future mortality rates?

- life assurance actuaries
- pension actuaries
- governments
  - Social security systems
  - Health and Care

3 main approaches

Expectation	Extrapolation	Explanation

Projecting by Expectation

Projections based on Expectation

Method historically favoured by government statistical agencies

Idea is to set an ‘expectation’ of future declines in mortality rates

2 main ways

- assume continuation of recent year rate of change
- convene group of experts to set a target for future reductions

Setting targets with Reduction Factors

Define $R_{x,t}$ the proportion by which q_x expected to reduce by some future year t

So if $q_{x,0}$ is the mortality rate at age x now at $q_{x,t}$ is the rate in future year t then

$$q_{x,t} = R_{x,t}\, q_{x,0}$$

Where $R_{x,t}$ is of the form

$$R_{x,t} = a_x + (1 - a_x)(1 - f_{n,x})^{t/n} \quad \text{for } 0<n<t$$

with

a_x = the ultimate reduction factor, and

$f_{n,x}$ = the proportion of decline occurring in n<t years

This gives an exponential shape to the decline in q_x rather than a linear one

Advantages & Disadvantages

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- easy to understand and implement

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- historically these methods have **underestimated** mortality improvements
- change in smoking
- improvements in prevention and treatment of heart disease and strokes
- problems of equating “targets” with “forecasts”

Possible developments

Use of large sample size questionnaires on health and wellbeing

- possible that self-reported health status more valuable than expert targets
- care needed that sample reflects population health factors
- potential data science application

Projecting by Extrapolation

Stochastic versus Deterministic approach

Methods based on expectation are essentially deterministic with fixed parameters inserted

For most investigations today, stochastic approaches are preferred – models that depend on a number of factors

This underlies the extrapolation based forecasting methods

The idea is that given some rates of mortality based on some (recent) past time period, we attempt to estimate rates of mortality for some future time period on the basis of one or more factors and time t.

Factors that apply to mortality forecasts

1. **Age** (x)
2. **Period** (t)
3. **Cohort** year of birth

Note that Cohort is not independent of Age and Period

Extrapolation methods create models of how mortality rates change over time with 1, 2 or 3 of these factors using past factor and mortality data to fit the model

The Lee-Carter Model

The Lee-Carter model

One of the most widely used models in forecasting future mortality rates

Two factor extrapolation model using Age + Period

Lee & Carter (1990s) wrote the original model in terms of **central rate of mortality** at some future time t denoted $m_{x,t}$

- See week 2 notes for definition of m_x and relation to q_x

$$\log m_{x,t} = a_x + b_x k_t + \varepsilon_{x,t}$$

Lee-Carter model (continued)

$\log m_{x,t} = a_x + b_x k_t + \varepsilon_{x,t}$

Where

a_x describes the general shape of mortality at age x

k_t measures the general population change in mortality in period to time t

b_x is the rate of response to the k_t change for age x

$\varepsilon_{x,t}$ are set of independent $N(0, \sigma^2)$ variables where σ is to be estimated

to find a, b, k, ε parameters some constraints are needed and usual ones are

$\sum_x b_x = 1$ and $\sum_t k_t = 0$

Estimation in the Lee-Carter model

estimate a_x as time average log mortality and then use matrix algebra methods

use maximum likelihood subject to constraints (the `gnm` package in R can be used)

Using the Lee Carter model for forecasts

The model has three parameters, two depending on age x and one (k_t) on time

Forecasting mortality then involves forecasting k_t whilst keeping a and b constant

The natural way to do this is by **Time Series** methods

- Random walk models
- Moving average models
- Auto regressive models

You will learn more about these in module MTH6139 next Semester

The key advantage of Lee Carter is that it is straightforward to apply using Time Series techniques once parameters have been estimated

Disadvantages of Lee Carter

- forecasts lose the smoothness of an original graduation over time
- using random walk time series methods can lead to relationships between mortality at different ages which we know is not reflected in practice
- there is no Cohort term, only Age and Period
  - we know from other investigations that UK mortality does exhibit Cohort effects over time (e.g. related to smoking patterns, wartime, COVID-19)
- future estimates become heavily dependent on the a_x and b_x parameters which are generally estimated from past data which might become less reliable as a guide to the future

Some forecasters have adapted Lee Carter to a 3 factor model including Cohort – we will not cover these in detail in this module

Splines and p-splines

We introduces Splines last week as a method of graduation

- polynomials of a specified degree joined piecewise at knots

It is also possible to use Splines in forecasting future mortality

- define log $E(D_x)$ as a polynomial function
- the more knots we have the better the combined spline function will adhere to past mortality data but the less smooth the result will become
- p-splines is a method for optimising the number of splines and knots
- this can be done with the `MortalitySmooth` package in R

Projecting by Explanation

Factors affecting mortality

The expectation and extrapolation methods do not use scientific knowledge on the factors affecting mortality and on medical advances

E.g. developments in cancer detection and treatment

However:

- modelling medical risk factors is as complicated as modelling mortality
- the time series data is not always of great quality or length
- cause of death data is not always reliable worldwide

Sources of error

Mortality forecasts are always wrong!

- Survival Model badly specified
- Uncertain parameter estimates
- Data set does not reflect population
- Climate change
- Random variation and data errors

Further Reading on Mortality Projections

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