MTH5129 Probability & Statistics II
Coursework 10 Solutions

1. Ten athletes ran a 400 metres race at sea level and at a later meeting the same athletes ran another 400 metres race at high altitude. Their times in seconds were as follows

Athlete	1	2	3	4	5	6	7	8	9	10
Sea level	48.3	47.6	49.2	50.3	48.8	51.1	49.0	48.1	50.7	47.9
High altitude	50.4	47.3	50.8	52.3	47.7	54.5	48.9	49.9	54.8	48.5

Using the matched pairs t-test, find the P value to see if the data provide any evidence to conclude that the athletes' performance is affected by altitude? (You should at least give the R command to find the P value but I would encourage you to find the value.) State the assumptions in applying the test. Calculate a 99% confidence interval for the mean difference in times at the two meetings.

Solution: The differences d_i are:

$$-2.1, 0.3, -1.6, -2.0, 1.1, -3.4, 0.1, -1.8, -4.1, -0.6$$

with sample mean $\bar{d} = -1.41$ and sample variance $s_d^2 = (1.645)^2$. The null hypothesis is $H_0 : \mu_d = 0$ and the alternative $H_1 : \mu_d \neq 0$. The test statistic is

$$T = \frac{\bar{d}\sqrt{n}}{s_d}, \qquad T \sim t_9 \text{ if } H_0 \text{ is true.}$$

Observed value of $T = \frac{-1.41\sqrt{10}}{1.645} = -2.71$. The p value is $2 \times P(T < -2.71)$. Using R we calculate

```
> 2*(pt(-2.71, 9))
[1] 0.0239975
```

So there is moderate evidence against H_0. It seems that the athletes performance is affected by the altitude. We assume the differences are normally distributed. In R we calculate

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> qt(.995, 9)
[1] 3.249836
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The form of a 99% confidence interval is

$$\bar{d} \pm t_{n-1}(0.995)\frac{s_d}{\sqrt{n}}.$$

$t_9(0.995) = 3.250$ So the 99% confidence interval is

$$-1.41 \pm 3.25 \times \frac{1.645}{\sqrt{10}} = (-3.101, 0.281).$$

2. Ten patients who suffered from insomnia were examined in a medical study to determine the effect of a sedative. Each patient received both the sedative and a placebo for a two-week period, the drugs being administered in random order, and there was a wash out period of one week in between the two two-week periods. Neither the patient nor the drug administrator knew which drug was being taken. The average number of hours sleep per night were recorded for each patient for each drug and the results were

Patient	1	2	3	4	5	6	7	8	9	10
Sedative	1.2	1.1	5.2	3.6	4.8	1.4	6.6	4.3	5.3	5.9
Placebo	0.6	1.1	3.5	2.8	2.9	2.0	3.7	3.7	5.5	5.2

a) Find the P value to test if there any evidence to show the sedative has a beneficial effect on patients by making them sleep longer?

b) Find a 95% confidence interval for the mean difference

c) Comment on the good features of the experimental design.

Solution:

a) The measurements are made on the same patient so a matched pairs t test is appropriate. The differences d_i are:

$$0.6, 0.0, 1.7, 0.8, 1.9, -0.6, 2.9, 0.6, -0.2, 0.7$$

with sample mean $\bar{d} = 0.84$, sample variance $s_d^2 = 1.123$ and sample standard deviation $s_d = 1.060$.

The null hypothesis is $H_0 : \mu_d = 0$ and the alternative $H_1 : \mu_d > 0$.

The test statistic is

$$T = \frac{\bar{d}\sqrt{n}}{s_d}, \qquad T \sim t_9 \text{ if } H_0 \text{ is true.}$$

Observed value of $T = \frac{0.84\sqrt{10}}{1.060} = 2.506$. The P value is $P(T > 2.506)$. Using R we calculate

```
> 1-pt(2.506, 9)
[1] 0.01676505
```

So there is moderate evidence against H_0. It seems that the sedative may have a beneficial effect..

b) Using R we calculate

```
> qt(.975, 9)
[1] 2.262157
```

The form of a 95% confidence interval is

$$\bar{d} \pm t_{n-1}(0.975)\frac{s_d}{\sqrt{n}}.$$

$t_9(0.975) = 2.262$

So the 95% confidence interval is

$$0.84 \pm 2.262 \times \frac{1.060}{\sqrt{10}} = (0.082, 1.598).$$

c) Good points of the design are:

- Use of a paired design which makes comparison more precise when patients are very variable.
- Use of a placebo to remove bias.
- Double blinding to remove bias.
- Drug administered in random order to remove bias.
- Use of a washout period to minimise carry-over or interaction effects.

3. Say whether you should use a two sample t-test or a matched pairs t-test in the following situations with a brief justification.

   a) Thirty people were weighed then randomly divided into two groups, each group was given a different diet. After two months the decrease in weight of each person was found.

   b) Fifteen pairs of twins were weighed. One of each pair were randomly allocated to group A the other to group B. Each group was given a different diet. After two months the decrease in weight of each person was found.

   c) Thirty people were weighed, they were arranged in weight order. One of the two heaviest people was allocated at random to group A the other to group B. Then the next two heaviest were similarly allocated and so on. Each group was given a different diet. After two months the decrease in weight of each person was found.

   d) Thirty people were weighed then arranged into pairs at random. One of each pair was randomly allocated to group A the other to group B. Each group was given a different diet. After two months the decrease in weight of each person was found.

Solution:

a) Two sample t test as there is no matching.

b) Matched pairs t test as the twins are matched genetically.

c) Matched pairs t test as the people are matched by their initial weight.

d) Two sample t test as the pairs are selected at random, there is no matching.

4. A standardized procedure for determining a person's susceptibility to hypnosis is the Stanford Hypnotic Susceptibility Scale (SHSS). This scale classifies a person's hypnotic susceptibility as Low, Medium, High or Very High. Researchers gave this test to a random sample of 130 undergraduates. The results classified by faculty of student were as follows:

	SHSS level				
	Low	Medium	High	Very High	Total
Arts	16	15	24	5	60
Science	25	20	19	6	70
Total	41	35	43	11	130

a) Determine the expected frequencies if there is no association between the two variables of classification.

b) Calculate the value of X^2 and hence test the hypothesis that there is no association at the 5% significance level.

c) What would have been the effect on your analysis if the above data had arisen from two independent samples, one of 60 Arts students and the other of 70 Science students?

Solution:

a) The expected frequencies are

	Low	Medium	High	Very High	Total
Arts	18.92	16.15	19.85	5.08	60
Science	22.08	18.85	23.15	5.92	70
Total	41	35	43	11	130

b) The null hypothesis is H_0 There is no association (independence) between faculty and hypnotic susceptibility versus H_1 there is an association. We find

$$X^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 2.61 \qquad X^2 \sim \chi^2_3 \text{ if } H_0 \text{ is true}$$

Using R we calculate

```
> qchisq(0.95,3)
[1] 7.814728
```

Therefore, the rejection region is $\{X^2 : X^2 > 7.815\}$. So we cannot reject the null hypothesis at the 5% significance level.

c) We would be testing a hypothesis of similarity of the distributions of susceptibility for Arts and Science students but the calculations would be unchanged.

5. In a study to examine different attitudes to healthy eating random samples of 747 men and 434 women were selected. Of those sampled 276 men and 195 women said they regularly order a vegetarian meal in a restaurant.

   a) Test the hypothesis that the proportions of men and women who order vegetarian meals regularly are the same against a two sided alternative, use a Z test and a significance level $\alpha = 0.01$.
   b) Find a 98% confidence interval for the difference in proportions.
   c) Find the corresponding 2×2 table and test the hypothesis that the proportions are the same using a chi-squared test. Confirm that your answer agrees with that in (a).

Solution:

a) We have the proportions of men and women ordering vegetarian meals as

$$\hat{p_1} = \frac{276}{747} = 0.369 \qquad \hat{p_2} = \frac{195}{434} = 0.449$$

The estimate of the overall proportion of people is

$$\hat{p} = \frac{276 + 195}{747 + 434} = \frac{471}{1181} = 0.399$$

We are testing $H_0 : p_1 = p_2$ against $H_1 : p_1 \neq p_2$.

The test statistic is

$$Z = \frac{\hat{p_1} - \hat{p_2}}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

We know that $Z \sim N(0,1)$ if H_0 is true.

The observed z is

$$\frac{0.369 - 0.449}{\sqrt{0.399 \times 0.601\left(\frac{1}{747} + \frac{1}{434}\right)}} = -2.71$$

Using R we calculate

```
> qnorm(0.995)
[1] 2.575829
```

The rejection region is $\{z : |z| > 2.5758\}$ so we reject H_0 at the 1% significance level and conclude that men and women do differ in ordering vegetarian meals.

b) Using R we calculate

```
> qnorm(0.99)
[1] 2.326348
```

A 98% confidence interval is of the form

$$\hat{p_1} - \hat{p_2} \pm 2.3263\sqrt{\frac{\hat{p_1}(1-\hat{p_1})}{n_1} + \frac{\hat{p_2}(1-\hat{p_2})}{n_2}}$$

so substituting the values

$$0.369 - 0.449 \pm 2.3263\sqrt{\frac{(0.369)(0.631)}{747} + \frac{(0.449)(0.551)}{434}} = (-0.149, -0.011)$$

c) The 2×2 table is

	Yes	No	Total
Men	276	471	747
Women	195	239	434
Total	471	710	1181

We test H_0 : Men and Women have the same proportions against H_1 : Men and Women have different proportions.

The test statistic is X^2 and $X^2 \sim \chi^2_1$ if H_0 is true.

The observed X^2 is (using the formula given in lectures)

$$\frac{(276 \times 239 - 195 \times 471)^2 1181}{747 \times 434 \times 471 \times 710} = 7.297.$$

Note we could have also found the expected frequencies as $\frac{747 \times 471}{1181}$ etc and calculated X^2 using its usual formula. Using R we calculate

```
> qchisq(0.99,1)
[1] 6.634897
```

The rejection region is $\{X^2 : X^2 > 6.635\}$ so we reject H_0 at the 1% significance level.

We note that $\sqrt{7.297} = 2.701$ which, apart from rounding error, is in agreement with the observed value of z in (a).

You are given the following R output.

```
> qt(.975, 10)
[1] 2.228139
> qt(.975, 9)
[1] 2.262157
> qt(.995, 10)
[1] 3.169273
> qt(.995, 9)
[1] 3.249836
> pt(-2.71,9)
[1] 0.01199875
> pt(2.506,9)
[1] 0.9832349
> qchisq(0.95,3)
[1] 7.814728
> qnorm(0.995)
[1] 2.575829
> qnorm(0.99)
[1] 2.326348
> qchisq(0.99,1)
[1] 6.634897
```