## MTH5129 Probability \& Statistics II Coursework 10 Solutions

1. Ten athletes ran a 400 metres race at sea level and at a later meeting the same athletes ran another 400 metres race at high altitude. Their times in seconds were as follows

| Athlete | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sea level | 48.3 | 47.6 | 49.2 | 50.3 | 48.8 | 51.1 | 49.0 | 48.1 | 50.7 | 47.9 |
| High altitude | 50.4 | 47.3 | 50.8 | 52.3 | 47.7 | 54.5 | 48.9 | 49.9 | 54.8 | 48.5 |

Using the matched pairs t-test, find the P value to see if the data provide any evidence to conclude that the athletes' performance is affected by altitude? (You should at least give the R command to find the P value but I would encourage you to find the value.) State the assumptions in applying the test. Calculate a $99 \%$ confidence interval for the mean difference in times at the two meetings.
2. Ten patients who suffered from insomnia were examined in a medical study to determine the effect of a sedative. Each patient received both the sedative and a placebo for a two-week period, the drugs being administered in random order, and there was a wash out period of one week in between the two two-week periods. Neither the patient nor the drug administrator knew which drug was being taken. The average number of hours sleep per night were recorded for each patient for each drug and the results were

| Patient | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sedative | 1.2 | 1.1 | 5.2 | 3.6 | 4.8 | 1.4 | 6.6 | 4.3 | 5.3 | 5.9 |
| Placebo | 0.6 | 1.1 | 3.5 | 2.8 | 2.9 | 2.0 | 3.7 | 3.7 | 5.5 | 5.2 |

a) Find the P value to test if there any evidence to show the sedative has a beneficial effect on patients by making them sleep longer?
b) Find a $95 \%$ confidence interval for the mean difference
c) Comment on the good features of the experimental design.
3. Say whether you should use a two sample $t$-test or a matched pairs $t$-test in the following situations with a brief justification.
a) Thirty people were weighed then randomly divided into two groups, each group was given a different diet. After two months the decrease in weight of each person was found.
b) Fifteen pairs of twins were weighed. One of each pair were randomly allocated to group A the other to group B. Each group was given a different diet. After two months the decrease in weight of each person was found.
c) Thirty people were weighed, they were arranged in weight order. One of the two heaviest people was allocated at random to group A the other to group B. Then the next two heaviest were similarly allocated and so on. Each group was given a different diet. After two months the decrease in weight of each person was found.
d) Thirty people were weighed then arranged into pairs at random. One of each pair was randomly allocated to group A the other to group B. Each group was given a different diet. After two months the decrease in weight of each person was found.
4. A standardized procedure for determining a person's susceptibility to hypnosis is the Stanford Hypnotic Susceptibility Scale (SHSS). This scale classifies a person's hypnotic susceptibility as Low, Medium, High or Very High. Researchers gave this test to a random sample of 130 undergraduates. The results classified by faculty of student were as follows:

|  | SHSS level |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
|  | Low | Medium | High | Very High | Total |
| Arts | 16 | 15 | 24 | 5 | 60 |
| Science | 25 | 20 | 19 | 6 | 70 |
| Total | 41 | 35 | 43 | 11 | 130 |

a) Determine the expected frequencies if there is no association between the two variables of classification.
b) Calculate the value of $X^{2}$ and hence test the hypothesis that there is no association at the $5 \%$ significance level.
c) What would have been the effect on your analysis if the above data had arisen from two independent samples, one of 60 Arts students and the other of 70 Science students?
5. In a study to examine different attitudes to healthy eating random samples of 747 men and 434 women were selected. Of those sampled 276 men and 195 women said they regularly order a vegetarian meal in a restaurant.
a) Test the hypothesis that the proportions of men and women who order vegetarian meals regularly are the same against a two sided alternative, use a Z test and a significance level $\alpha=0.01$.
b) Find a $98 \%$ confidence interval for the difference in proportions.
c) Find the corresponding $2 \times 2$ table and test the hypothesis that the proportions are the same using a chi-squared test. Confirm that your answer agrees with that in (a).

You are given the following $R$ output.

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> qt(.975, 10)
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[1] 2.228139
> qt(.975, 9)
[1] 2.262157
> qt(.995, 10)
[1] 3.169273
> qt(.995, 9)
[1] 3.249836
> pt $(-2.71,9)$
[1] 0.01199875
$>\operatorname{pt}(2.506,9)$
[1] 0.9832349
> qchisq(0.95,3)
[1] 7.814728
> qnorm(0.995)
[1] 2.575829
> qnorm(0.99)
[1] 2.326348
> qchisq(0.99,1)
[1] 6.634897

