QUEEN MARY, UNIVERSITY OF LONDON

MTH6102: Bayesian Statistical Methods

Practical 11

2023-2024

Symmetric Metropolis-Hastings algorithm implementation issues

Binomial data/beta prior

The R code file "practical 11 R code beta-binomial.R" implements the symmetric Metropolis-Hastings algorithm for the binomial example with beta prior distribution that we saw in the lecture 9B

- Data: $k = 12 \sim \text{binomial}(40, q)$, where q is the probability of success.
- Prior: $p(q) \sim \text{beta}(2, 2)$.

Our goal is to construct an approximated sample $q_1, q_2 \ldots q_M$ from the posterior density, which is beta$(14, 30)$. In this lab, we will examine the effect of the standard deviation b of the normal proposal distribution on the behavour of the algorithm.

Try changing b, which is the standard deviation of the proposal distribution (a normal distribution). Change it to a small (positive) value such as 0.001, and then a large value such as 10. What is the effect on how the algorithm behaves of these different values for b? Plot the vector of q values against the iteration number to visualize the behaviour, as in the lecture code.

Rerun the algorithm using values of $b = 0.001, \ldots, b = 10$, and each time calculate the proportion of steps that were accepted. Then plot this acceptance probability against b. Examine how the acceptance probability for this algorithm depends on b. Which value of b gives an acceptance probability between 20%-30%?

Rerun the algorithm using a value of b such that the acceptance probability is between 20%-30%. Plot the histogram of the sample and the exact beta posterior density on the same graph. Check that summaries of the posterior distribution are similar to what you would expect for the exact beta posterior.