Tutorial question on Graduation – solution

IFoA CT4 April 2017 Q10

(i) we graduate here because:

- we believe mortality rates proceed smoothly with age
- sampling errors exist within the data
- the survival model would have been run separately for each individual age
- graduation allows us to use data from neighbouring ages
- ongoing use of a new standard table is more acceptable if rates are smoothed

(ii) graduation by some parametric formula is best here (graphical methods are possible but inferior, graduation with reference to an old standard table does not make sense for a new one)

(iii) limitations include:

- more difficult where there is little data (very oldest ages)
- any errors / bias in the data cannot be eliminated
- problems with any extrapolation for ages with no data

(iv) we will complete the standard deviations and the signs test here

Standard Deviations test:

the null hypothesis is that the graduated rates reflect the crude mortality estimates

Range	*< -2*	*(-2,-1)*	*(-1,0)*	*(0,1)*	*(1,2)*	*> 2*		*sum*
Actual	7	16	26	16	10	5		80
Expected %	2%	14%	34%	34%	14%	2%		
Expected	1.6	11.2	27.2	27.2	11.2	1.6		
(A-E)^2/E	18.23	2.06	0.05	4.61	0.13	7.23		32.30

our chi-squared statistic is 32.30

we compare this to the upper 5% critical value of the chi-sq distribution on 5 degrees of freedom (6 ranges minus 1) which = 11.07 < 32.30

therefore we reject the null hypothesis at 95% significance – the graduation does not reflect the crude mortality estimates

Signs test:

the null hypothesis is that the graduated rates reflect the crude mortality estimates

we have 49 negative z_x's and 31 positive ones. under the null hypothesis the number of negative values ~ Binomial(80, ½) and with n=80 we can use the Normal approximation of the Binomial where the number of negative values ~Normal(80 x ½, 80x ½ x ½) = N(40,20)

our z-score = (49 – 40) / √20 = 2.012 which is > 1.96 critical value at 95% (two tailed test) so we do not reject the null hypothesis and on the basis of the signs test the graduated rates do reflect the crude estimates.

(v) from these two tests:

- the null hypothesis is rejected by the standard deviations test and (marginally) not rejected by the signs test at the 95% significance level
- the standard deviations test indicates the presence of some large individual deviations or outliers
- this means use of the graduated table for government purposes would cause some concern
- for regulatory purposes we would prefer a table with better adherence to experience data
- the overall overestimation of mortality by the graduation (more negative individual deviations) would incur financial risk if the table was used in national pensions calculations