Random Processes – 2023/24 Assessment 2

This assessment consists of three questions. The focus is particularly on exam preparation. This does not mean that these are practice exam questions (in-term assessment is not meant to test skills in the same way as exams). Instead, it covers skills for getting yourself exam-ready.

- Summarising (Question 1)
- Navigating your notes (Question 2)
- Adapting your knowledge to deal with the unexpected (Question 3)

Please submit your solutions to all questions via the module QMplus page. Work should be submitted as a handwritten and scanned (or electronically written on a tablet) document. The submission item in the Week 11 topic of the QMplus page has more details and you should following the instructions there.
You must submit your work by **5:00pm on Friday 8 December 2023**. Late work will not be accepted.
The work you submit must be your own. It is fine to discuss the problems with other students but you must write up your solutions yourself. Copying work from another person or getting someone else to do your work constitutes an assessment offence.

Further Instructions and Guidance

As always, read each question carefully and make sure that you answer the question you are asked and that your answer is in the form required (this is another good exam skill).

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1. (40%) This question is about summarising a section of the lecture notes. Your answer to Question 1 must consist of 1 side of A4 in normal sized writing. Marks will be given for being concise, mathematically accurate, and covering most of the topic.

(a) Make a summary of the topic Poisson processes (chapter 7 of the notes) as discussed in week 9 seminars. You could do this as a concept-map (visual summary) or an organised list of key points[1].

(b) State which single aspect of this topic needed the most help from your notes.

2. (40%) This tests how well you can identify results and concepts from the module based on a description of the idea behind them or how they might be used. You will need to refer to a copy of the online notes from the module QMplus page.

Each part asks for a definition or a result (Theorem, Lemma etc.) and your answer should be in the following form[2]:

- Definitions: the number of the definition and name the of the concept defined (eg Definition 23 – birth process).
- Results: the number of the result (eg Theorem 8.1).

In chapters 1 to 7 of the notes, find:

(a) A definition which captures the idea of a process getting stuck somewhere for ever.

(b) A result which involves ignoring certain arrivals in a Poisson process.

(c) A definition of a method of representing a Markov chain in a way that allows linear algebraic methods to be applied.

(d) A result which would be useful to know if you wanted to prove that null recurrence is a class property.

(e) A definition of something which could be used to give an alternative definition of the property of being irreducible.

[1] To get the most out of this, try the excerise first without looking at your notes. Then compare with your notes to see what you have missed out or not got quite right. You should submit only a single final version of your summary.

[2] The examples use items from later in the notes to avoid giving anything away.

(f) Two results related to giving alternative definitions of the Poisson process equivalent to the one we gave as Definition 21.

(g) A result whose proof involves constructing a regular Markov chain from an irreducible Markov chain in such a way that the equilibrium distributions do not change.

(h) A result which would help find the probability that we reach state a before state b.

(i) A result that says how to calculate the probability of following a particular trajectory through the state space.

3. (20%) Read the following and answer the questions about it. The passage describes a particular type of process we do not study in this module but the concepts and skills you have learnt in this module will help you digest it.

> This continuous-time process with state space $\mathbb{N}$ starts in state 0 and repeats in the following way. It remains in the same state for some random *waiting time* at which point the state number increases by some random *increment*. The waiting times are independent and all follow the same fixed probability distribution taking values in $\mathbb{R}_{\geqslant 0}$. The increments are independent (and also independent of the waiting times) and all follow another fixed probability distribution taking values in $\{1, 2, 3, \ldots\}$.
> So, if the waiting times are $S_1, S_2, \ldots$ and the increments are $n_1, n_2, \ldots$ then we start with $X(0) = 0$ and remain with $X(t) = 0$ for $t \in [0, S_1)$. Then we jump up by n_1 so $X(t) = n_1$ for $t \in [S_1, S_1 + S_2)$. Then, at time $S_1 + S_2$, we jump up to $n_1 + n_2$ and so on. More formally, for $t \in [S_1 + \cdots + S_k, S_1 + \cdots + S_k + S_{k+1})$ we have $X(t) = n_1 + \cdots + n_k$.

(a) Which choices of distribution for waiting time and increment show that the Poisson Process is a special case of this kind of process?

(b) You want to model customers arriving in a shop using this process instead of a Poisson Process. Using a random variable to determine the increments allows us to include a feature which the Poisson Process cannot. Write down this feature in non-mathematical language.

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