

QUEEN MARY, UNIVERSITY OF LONDON

MTH6102: Bayesian Statistical Methods

Practical 9

2023-2024

This lab is a review of what we have covered so far and it will help you for the exercise sheet 8 and the final exam. There will be no solutions but you can ask us during the IT class.

Review

In a factory producing computer chips, on the production line chips are inspected one at a time until a defective chip is found. The probability of each chip being defective is q , where q is unknown. The number of chips inspected before the first defective one is found can be modelled using the geometric distribution with parameter q , and pmf

$$p(x|q) = (1 - q)^x q, \quad x = 1, 2, \dots$$

This process is repeated n times, and the number of chips inspected before the first defective one was x_1, \dots, x_n . Let that the observed data for $n = 5$ be $x_1, \dots, x_5 = 6, 2, 7, 0, 4$.

1. What is the maximum likelihood estimate (MLE), \hat{q} , of q ?
2. A $\text{beta}(\alpha, \beta)$ prior distribution is chosen for q . Before seeing the data, we would like the prior mean to be 0.2 and the standard deviation 0.1. Find the parameters of a beta distribution that satisfy this.
3. In R, generate a random sample of size 10,000 from the beta distribution derived as a prior in question (b), and check that the mean and standard deviation of the sample are approximately 0.2 and 0.1, respectively.
4. Find the posterior distribution for q under the geometric likelihood and the prior computed from (b). What is the posterior mean?
5. Show that the posterior mean for q is always in between the prior mean and the MLE for this example. As $n \rightarrow \infty$ show that the posterior mean tends to the MLE of q .
6. Use R to find the posterior median and a 95% credible interval for q .

Let $\psi = E(q^2)$, be the second moment of q .

7. Use Monte Carlo integration to approximate ψ and plot the histogram of an IID sample from ψ .