QUEEN MARY, UNIVERSITY OF LONDON

MTH6102: Bayesian Statistical Methods

Exercise sheet 8

2023-2024

This is assessed and counts for 4% of the module total. The deadline for submission is Monday the **27th November at 11am**.

Submit the R code used as an R script file (with extension .R) or screenshot. But you need to write the answers and report the R output in a separate file. This can be a pdf document or a clearly legible image of hand-written work.

1. **10 marks.** For the geometric model of exercise sheet 5, question 2, suppose that for a prior distribution, we assign a Beta(α, β) distribution. We take $\alpha = 1$ and choose β so that the prior probability $P(q \leq 0.4)$ is 0.9.

   (a) Find the value of β that satisfies this requirement.

   (b) Use R to simulate a large sample from this beta distribution, and so check your calculation.

2. **45 marks.** Let $x_1, \ldots, x_n$ iid from $N(\mu, \sigma^2)$ where $\sigma^2 = 1$.

   (a) Show that the Jeffreys prior for the normal likelihood is

   $$p(\mu) = c_1 \sqrt{n/\sigma^2}, \quad \mu \in \mathbb{R}$$

   for some constant $c_1 > 0$.

   (b) Is this a proper prior or improrer prior? Explain.

   (c) Derive the posterior density for μ under the normal likelihood $N(\mu, \sigma^2)$ and Jeffreys prior for μ. Plot the density. Use the same dataset `x` that you used in problem 2, exercise sheet 5.

   (d) Simulate 1,000 draws from the posterior derived in (c) and plot a histogram of the simulated values.

   (e) Let $\theta = \exp(\mu)$. Find the posterior density of θ analytically and plot the density.

   (f) Estimate θ by Monte Carlo integration.

   (g) Compute a 95% equal tail interval for θ analytically and by simulation.

3. **45 marks.** Consider the 10 Bernoulli(q) observations: 0,1,0,1,0,0,0,0,0,0. Plot the posterior for q using these priors: Beta(0.5,0.5), Beta(1,1), Beta(10,10), Beta(100,100). Comment on the effect of the prior on the posterior.