Probability and Statistics II

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- You can find me at the Learning Café on Thursdays just before the lecture (12-1)

- Assessment worth 10% in week 12 and in-person exam in January
- Structure of the Statistics part of the exam
- Formula sheet is available on the QMplus page!
- Some use of R in this part of the module

A company uses a machine to produce bags of crisps. On the packaging, the advertised weight of a bag is 50g. Three randomly selected bags are found to have mean weight of 49g. Can you conclude that the machine is not working correctly?

A company uses a machine to produce bags of crisps. On the packaging, the advertised weight of a bag is 50g. Three **thousand** randomly selected bags are found to have mean weight of 49g. Can you conclude that the machine is not working correctly?

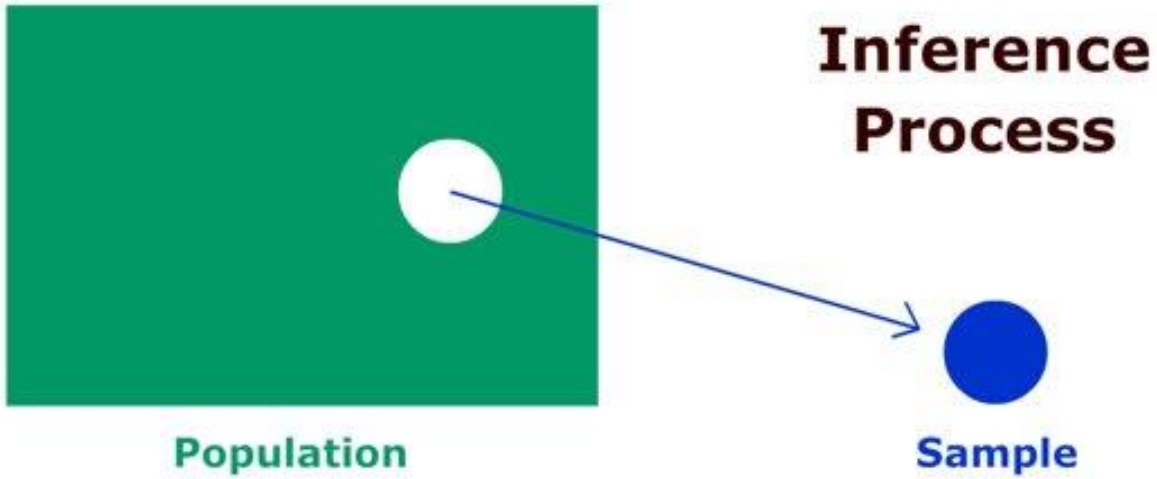
In this module we will learn how to answer these statistical questions!

We will cover:

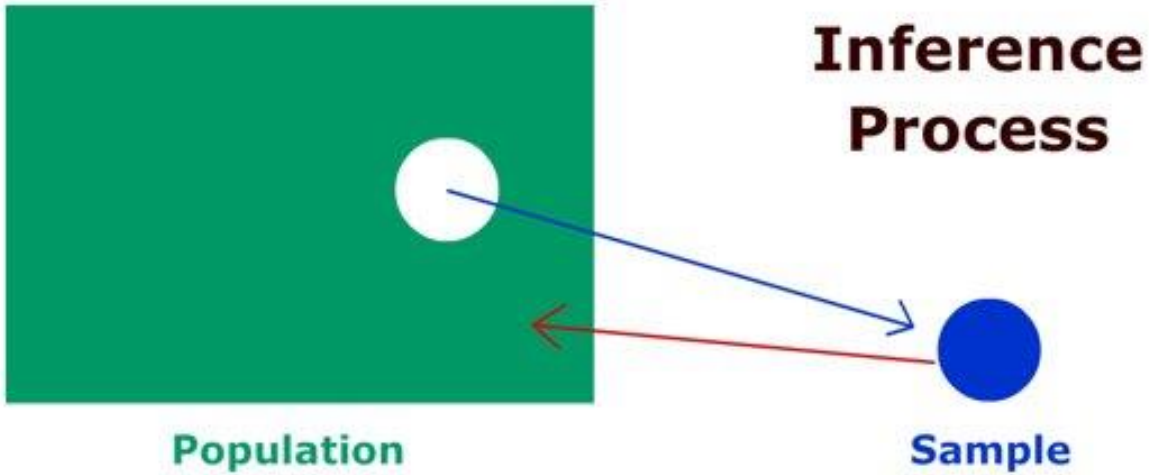
- Confidence intervals
- Hypothesis testing

Inference Process

1. From a **Population**, take a sample.



1. From a **Population**, take a sample.
2. Analyze the **Sample**.

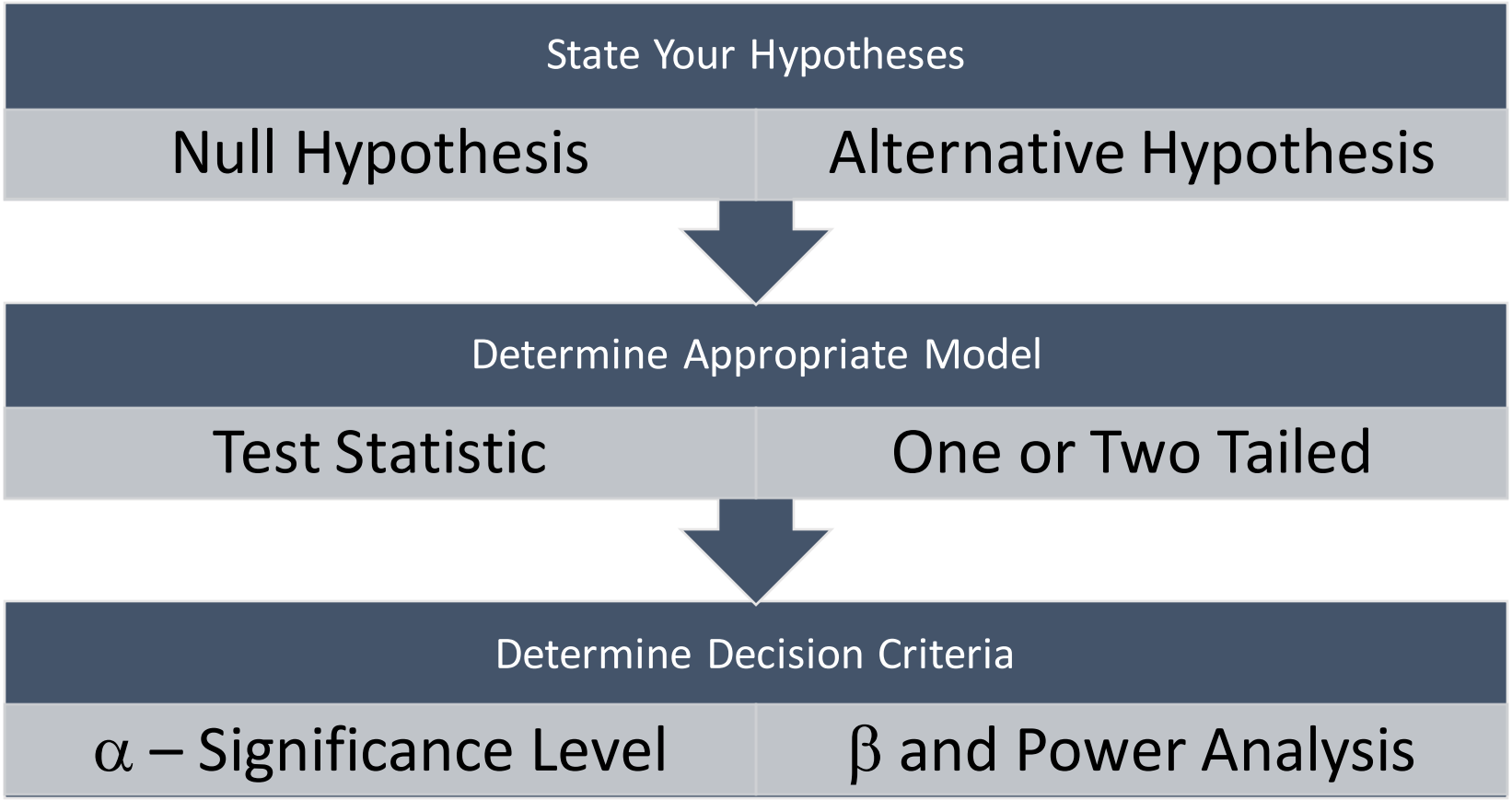


1. From a **Population**, take a sample.
2. Analyze the **Sample**.
3. Make an **Inference** about the Population based on the Sample.

What is Hypothesis Testing?

- Hypothesis testing: A procedure, based on sample evidence and probability theory, used to determine whether the hypothesis is a reasonable statement and should not be rejected, or is unreasonable and should be rejected.

Hypothesis Testing Design



Definitions

- Null Hypothesis H_0: A statement about the value of a population parameter that is assumed to be true for the purpose of testing.
- Alternative Hypothesis H_a: A statement about the value of a population parameter that is assumed to be true if the Null Hypothesis is rejected during testing.

Definitions

- Level of Significance: The probability of rejecting the null hypothesis when it is actually true. (signified by α)
- Type I Error: Rejecting the null hypothesis when it is actually true.
- Type II Error: Failing to reject the null hypothesis when it is actually false.

Outcomes of Hypothesis Testing

	Fail to Reject Ho	**Reject Ho**
Ho is true	Correct Decision	**Type I error**
Ho is False	**Type II error**	Correct Decision

EXAMPLE – Type I and Type II Errors

- Ho: The mean amount of Soy Sauce is 16 ounces
- Ha: The mean amount of Soy Sauce is not 16 ounces.

- Type I Error: The researcher **supports** the claim that the mean amount of soy sauce is not 16 ounces when the actual mean is 16 ounces. The company needlessly “fixes” a machine that is operating properly.

- Type II Error: The researcher **fails to support** the claim that the mean amount of soy sauce is not 16 ounces when the actual mean is not 16 ounces. The company fails to fix a machine that is not operating properly.

Definitions

- Critical value(s): The dividing point(s) between the region where the null hypothesis is rejected and the region where it is not rejected. The critical value determines the decision rule.

- Rejection Region: Region(s) of the Statistical Model which contain the values of the Test Statistic where the Null Hypothesis will be rejected. The area of the Rejection Region = α

The logic of Hypothesis Testing

- This is a "Proof" by contradiction.
  - We assume H_o is true before observing data and design H_a to be the complement of H_o.
  - Observe the data (evidence). How unusual are these data under H_o?
  - If the data are too unusual, we have "proven" H_o is false: Reject H_o and go with H_a (Strong Statement)
  - If the data are not too unusual, we fail to reject H_o. This "proves" nothing and we say data are inconclusive. (Weak Statement)
  - We can never "prove" H_o , only "disprove" it.
  - "Prove" in statistics means support with the Alternative Hypothesis.
  - Note: It is **never correct** to say $(1-\alpha)100\%$ certain of our decision. (example: if $\alpha=.05$, then we are not 95% certain if we Reject Ho.)

Review: R commands