Introduction

This handout contains some advice on revision and exam technique aimed particularly at the Random Processes exam. You may find it useful for other modules but other module organisers may have different views on exams and revision and you should always follow their advice if it differs from this.

What to Expect from the Final Exam

You can expect the final exam to contain:

- A few (4 or 5) multi-part questions in a broadly similar style to problem sheet questions.
- A mix of easier and harder questions (it should be very difficult to get over 90%)
- Questions consisting of a mix of calculations, explanations, proofs, etc.
- A limited number of bookwork questions (eg State some definition or state and prove some result from the notes[1]).
- Some unfamiliar looking questions. Everything will be based on the lectured material but you may need to tweak a method slightly to use it in a situation which is not quite the same as our examples or problem sheet questions.
- Questions covering a good portion of the lectured material. Some topics might not come up, but there will be questions on a good spread of topics from across the module (for instance it won't be the case that every question is about continuous-time processes).

[1] I don't expect you to remember the Theorem numbers from the notes. Anything will be referred to either by name (eg "The Chapman-Kolmogorov Relations' or "The Thinning Lemma for Poisson Processes") or description (eg "A Theorem giving the first-step analysis equation for absoption probabilities").

Different Ways to Test Knowledge

There are lots of different ways to test knowledge and skills.

Suppose I want to find out whether you know and can use the definition of an equilibrium distribution of a discrete-time Markov chain. I could ask you to:

- State the definition.
- Find the equilibrium distribution of a Markov chain given by its transition matrix.
- Find the equilibrium distribution of a Markov chain given in some other way.
- Explain a method for finding the equilibrium distribution of a Markov chain.
- Check that a particular probability vector is the equilibrium distribution of a given Markov chain.
- Construct a Markov chain which has a particular probability vector as the equilibrium distribution.
- Prove a simple result about equilibrium distributions.
- Give an example showing that a simple statement about equilibrium distributions is false.

You need to be prepared whichever of these comes up. If your preparation has just consisted of practising finding the equilibrium distribution from the transition matrix then you will be fine if you happen to get a question of that form but might struggle if I choose a different way to test this knowledge. The key point is to focus on mastering the material rather than just particular processes. This means understanding the definitions and being able to use them in a variety of ways. The problem sheet questions, the mid-lecture tasks, and the lecture notes will all help with this.

> **Tip:** When revising think about concepts not procedures.

Preparation

The best way to be prepared for the exam is to work solidly through the term so that you are always on top of the material. If you have done this then revision will be a smooth and relatively simple process.

The problem sheet questions, are a key part of the module material. Every question on a problem sheet could have formed the basis of a legitimate exam question (of course some of them are harder or easier than others but that is true for exam questions too).

If you have attended all the lectures, diligently read the notes, and made a serious attempt at the problem sheets then you should be well on the way to mastering the material of the module. This means that, as well as being able to do questions that are in the same form as examples and problem sheet questions, you will also be able to do more challenging unseen questions.

Here is a good test you can apply to check if you have mastered a section of the material. Ask yourself: if someone were to name a concept or topic from the module would I be able give a summary of it (including definitions, informal explanations, some applications and examples, some theorems and proofs)? If the answer is 'yes' for most concepts from the module then you can genuinely say you have mastered the material and you will do well in the exam. Making this kind of summary without looking at your notes is a good thing to do as part of exam preparation. It can help you digest the material and embed it in your memory as well as acting as a test of what you know and what you need to spend more time on.

> **Tip:** Stay on top of things and check your understanding regularly.

Past Papers

It is good to have a go at past papers but recognise that just doing past papers is not a complete revision strategy.

In every year exam papers contain a mixture of bookwork, routine examples and unseen material. The unseen questions will vary from year to year; a good exam should always have a few surprises!

As described in the previous section, a good way to prepare is to master the material of the module rather than to just get good at the particular questions that were asked in previous years.

You will get the most from your attempts at past papers if you persist even if a question looks difficult, and you make a serious effort to check your answers using your own mathematical skills and lecture notes. Providing model solutions discourages both of these so I tend to do this in a limited way only. I'm always happy to offer a hint on any past paper question you get stuck on (probably in the student forum so everyone has seen the same information). I will post solutions to one or two past papers towards the end of term

The past papers available on the School repository contain papers set by different module organisers and sat under different conditions (online or on-campus). Generally, online exams will have fewer bookwork questions. However, even in an on-campus exam I am more interested in testing what you can do with the material then whether you can reproduce the notes. This means that I tend towards fewer bookwork parts and more questions asking you to explain things, or adapt a method to an unseen situation. The papers from January 2023 and January 2022 were both set by me and are probably the best guide to the style of question you can expect despite both being online exams. Earlier papers were set by different module organisers but are still relevant (the material covered has not changed sugnificantly).

Tip: Do some past papers but don't make this the only part of your preparation.

Just Before the Exam

You will do your best work if you are physically prepared. In the time running up to the exam take good care of your physical wellbeing (exercise, rest, diet). Don't leave all your revision to the last minute and make sure that you get plenty of sleep the night before the exam so that you go into it well rested and relaxed.[2]

Tip: Look after yourself so you can do your best work in the exam.

[2]Just as an athlete will 'taper' (ease off the intensity of their training) immediately before a big race.

In the Exam

During the exam, the main principles to remember are:

- Read the question . . . and answer it!
- Express yourself clearly and accurately.
- Convince the examiner that you know what's going on.

None of these are very profound, but under the pressure of exam stress it can be easy to not do them. Try to identify and other particular errors that you are know you often make. This could be specific mathematical things or more general aspects of exam technique. If you have thought about these in advance the you can be on your guard for them in the exam.

> **Tip:** Learn what kind of silly mistakes you are most prone to and consciously guard against them.

Read the question . . . and answer it!

The golden rule for exams is to read each question carefully and answer it as well as you can.

- Look out for words that instruct (State, Prove, Justify,. . .)
- Don't miss parts out.
- Or add extra parts (if you are just asked to state a Theorem don't give a proof as well).
- Answer the question asked and not a superficially similar one.
- Use any hints or implicit guidance (eg part (a) may help with part (b))

If a question asks for a definition then you must give a definition rather than an example; if a question asks you to prove something then you need to give a proof; if a question asks you to put the answer in a particular form then you need to do this. This sounds obvious but many people lose marks through not answering the question asked. Unlike problem sheet questions

which may be (intentionally or accidentally) open-ended or ambiguous, I will phrase exam questions carefully so that it is clear what is being asked for.

Don't panic if a question doesn't look quite as expected. One of things we are testing is how well you can use the skills and techniques you have learnt in slightly unfamiliar situations.

Express yourself clearly and accurately

Make sure that all your written work is legible and number your answers (questions and parts). When correcting something it is often better to re-write than amend (a + changed into a − can be hard to decipher for instance).

Many students use words too sparingly so remember that words matter! This could be:

- To give logical structure.

  We need to check whether $p_{0,0}^{(t)} \to 0$

  is not the same thing as

  From the condition given we have $p_{0,0}^{(t)} \to 0$

- To justify steps in an argument. Give a brief reason why each line of a proof is true (unless it is very obvious such as a simple maipulation) so it is clear to the examiner that you know what you are doing. These can be very succinct: "Since A and B are disjoint", "Substituting equation (1)", "By the Markov property", "Because s is recurrent" etc.)

Convince the examiner

Keep in mind that your job is to convince the examiner that you know what you are doing.

When doing calculations, show your working and explain what you are doing. If you make a small mistake but the examiner can still see what you were trying to do then you may get some marks:

For example: Customers arrive in a shop as a Poisson process of rate 1/2. What is the probabilty that 2 customers arrive in the first 10 minutes:

- $\frac{25}{4e^5}$ Wrong! Would get 0 marks.

- $X(10) \sim \text{Po}(10/2) = \text{Po}(5)$ so $\mathbb{P}(X(10) = 2) = e^{-5}\frac{5^2}{2^2} = \frac{25}{4e^5}$ Still wrong but would get some marks

When writing proofs, give well-structured, concise arguments. Most people don't use words enough in proofs (as discussed above).

If you get stuck say what you are trying to do. If you think something is wrong but can't see what it is then say so. I would be well-disposed (possibly translating into marks) towards candidates who write this kind of thing:

- The answer will be the solution to the following equation but I can't see how to solve it.
- So $\mathbb{P}(A) = 5/4$. (I know this can't be right since probabilities are $\leqslant 1$ but I can't find my mistake!)

Checking Your Work

If you have time at the end of the exam then check your work. This could involve just reading over what you have written carefully. Do this critically, asking yourelf whether every assumption you have made is justified and every step is correct. Ask yourself whether the final answer makes sense and seems believable.

It is easy to fall into the same trap when reading an incorrect argument, so if possible try to find a different route to check an answer. For example: If you calculated an equilibrium distribution by solving a matrix equation then check your answer by substituting it back into the equation.

> **Tip:** Don't panic! Slow down. Read and write carefully and check your work.

Anything Else?

If you have questions on any aspects of the final exam not which are not covered here then let me know so I can add to this document.

Best of luck with the exam!

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