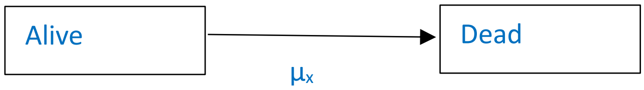
Tutorial Question week 5 Multi State Models

A life assurance company offers a one year term insurance with a sum assured of $20,000 payable immediately on death within the one year term in return for premiums which are payable weekly through the year. The company uses a two state model to estimate mortality needed for premium and reserve calculations and ignores both interest and expenses in those calculations.

(a) Draw and fully label the two state model diagram for this policy.



Where μ_x is the transition intensity from state alive to dead at age x

(b) Write down an expression for the likelihood of this model in terms of the transition intensity used in (a) above.

$L(\mu) = \exp(-\mu v)\ \mu^d$

Where the transition intensity μ_x is assumed to be a constant μ in a particular year of age

And d is the number of deaths observed, v is the observed waiting time or Central Exposed to Risk

In 2022 the insurance company issued 2423 policies to lives age 59. Of those people, one died on 31 March 2022 and another on 30 September 2022.

(c) Calculate the maximum likelihood estimate of the transition intensity in (b) using this data.

The MLE for μ is d/v

d=2

v= 2423 – ¾ – ¼ = 2422

so $\hat{\mu}$ = 2 / 2422 = 0.00826

(d) If 2022 data is used to calculate premiums charged in 2023, calculate the weekly premium for a new term insurance policy for a life age 59 on 1 January 2023, stating any assumptions you make.

We assume here that a weekly premium approximates to 1/52 of an annual continuously payable premium.

The equation of value is PV premiums = PV benefits

Let the annual premium continuously payable be P

PV benefits = 20000 $\hat{\mu}$ = 16.51528 ignoring interest

PV premiums = P $\int_0^1 {}_tp_{59}\ dt$ = P $\int_0^1 \exp(-\mu t)\ dt$ = P $\left[-\frac{\exp(-\mu t)}{\mu}\right]_0^1$ = 0.99959 P

Therefore P = 16.51528 / 0.99959 = 16.5221

And so the weekly premium = 0.317733 or $0.32

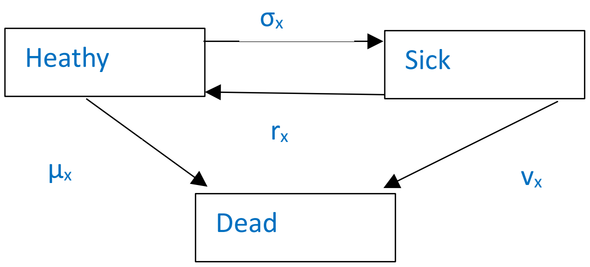
(e) What concerns might the insurance company have about using the premium calculated in (d) above?

The calculation only depends on μ_hat_59 which was estimated from one year's data and is very sensitive to the observed number of deaths (=2) One more or one fewer observed death in that year would lead to very different premium amounts.

The model has not taken any selection into consideration.

The company is considering introducing a new term insurance policy for 2024 which is the same as that above except that policyholders will not have to pay premiums whilst they are sick.

(f) Draw and label the new multi state model needed for this policy.



Where σ is the transition intensity from healthy to sick, r the transition intensity from sick to healthy, μ the transition intensity from healthy to dead and v the transition intensity from sick to dead.

(g) If the transition intensity from healthy to sick is 0.0054 and the transition intensity from healthy to dead is the same as that in (c) above, calculate the probability that a policyholder age 59 will pay the full 52 weeks of premium in the new policy arrangement.

Pr[pays full year premium] = Pr[remains in healthy state all year]

= exp(– {sum of the transition intensities out of that state})

= exp(– 0.0054 – 0.000826) = 0.993794