ODEs For Stars

1 Form of the ODEs to solve

There are N stars, labelled by an index $i = 1, 2...N$.

The position of star i is (x_i, y_i), its mass is M_i. The goal is to solve for the positions as functions of time, given some initial values of $(x_i(t=0), y_i(t=0))$.

Notice that this is a *2nd order ODE*, so we need to apply what we learned in the course about dealing with higher order ODEs. This also means that our initial conditions will need to include the first derivatives $(\dot{x}_i(t=0), \dot{y}_i(t=0))$, as well as the positions themselves.

The ODE for the x position of each star is:

$$\frac{d^2 x_i}{dt^2} = \sum_{j=1, j\neq i}^{j=N} \frac{GM_j}{r_{ij}^2} \cos(\theta) \tag{1}$$

and for the y position

$$\frac{d^2 y_i}{dt^2} = \sum_{j=1, j\neq i}^{j=N} \frac{GM_j}{r_{ij}^2} \sin(\theta) \tag{2}$$

where

- $r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$
- $\cos\theta = (x_j - x_i)/r_{ij}$
- $\sin\theta = (y_j - y_i)/r_{ij}$

2 Units and initial conditions

If you set $G = 1$, then you can work in units in which the masses of the stars are order 1 numbers, as are the values of r_{ij} (roughly speaking you are choosing the length units to be equal to the mass units). You can also choose to make the star velocities $\dot{x}_1, \dot{x}_2 \ldots$ order 1 numbers, in order to get orbital timescales of order 1 (roughly speaking you are choosing the length units to be equal to the time units). Note that what I call an order 1 number here is anything in the range from 0.1-10!

In some of the coursework you have to choose your own initial conditions - here you may need to do a bit of experimentation to get something that is sensible and has the behaviour that is asked for in the instructions.