QUEEN MARY, UNIVERSITY OF LONDON

MTH6102: Bayesian Statistical Methods

Exercise sheet 2

2023-2024

The deadline for submission is **Monday the 16th October at 11am**. Late submissions receive zero marks. You can submit a Word document, pdf or a clearly legible image of hand-written work.

(1) **10 points**. Let X be a discrete random variable with pmf $p(x|\theta)$, $\theta \in \{1, 2, 3\}$. One data point x is taken from $p(x|\theta)$. Find the MLE of θ.

x	$p(x \mid 1)$	$p(x \mid 2)$	$p(x \mid 3)$
0	1/3	1/4	0
1	1/3	1/4	0
2	0	1/4	1/4
3	1/6	1/4	1/2
4	1/6	0	1/4

(2) **20 points.** Let $Y_1, \ldots, Y_n$ be an iid sample from $N(\mu, \sigma^2)$, with both μ and σ^2 unknown.

(a) Find the likelihood and log likelihood functions.

(b) Find the maximum likelihood estimates $\hat{\mu}$ and $\hat{\sigma}$.

(3) **20 points.** In a certain factory, machines D, E and F all produce computer chips of the same type. Of their production, machines D, E and F, respectively produce 2%, 3% and 1% defective chips. Machine D produces 30% of the output of the factory, machine E 25% and machine F the rest.

Suppose one chip is selected at random from the output of the factory and the chip is defective

(a) Use Bayes' theorem to find the probabilities that the chip was manufactured on machines D, E and F.

(b) Identify the data, hypotheses, likelihoods, prior probabilities and posterior probabilities.

(c) Redo the computation of (a) using a Bayesian updating table.

(4) **50 points**. Suppose that you have recently started taking a train to work in a new location. You would like to estimate the probability q that the train arrives no more than 5 minutes late. Based on past experience living in South London, you assign a Beta distribution for q with parameters $\alpha = 5, \beta = 25$ as a prior distribution.

(a) What is the mean of the prior distribution?

Suppose that you observe k late arrivals in n journeys. For this observed data, use digits from your student ID number. Let the last three digits of your ID number be ABC. Then take $n = 10 + AB$ and $k = C$. (E.g. if the ID ends in ...092, then $n = 10 + 09 = 19, k = 2$; if the ID ends in ...374, then $n = 10 + 37 = 47, k = 4$)

(b) What is the maximum likelihood estimate $\hat{q}$ for q?

(c) What is the posterior distribution for q?

(d) What is the mean of the posterior distribution? What is the variance of the posterior distribution?

(e) What would the mean of the posterior distribution be if you had taken as a prior distribution the uniform distribution on $[0, 1]$?