Notes on survival model concepts and associated calculations using R

Chris Sutton, October 2023

These notes explain how R can be used for calculations involving force of mortality and survival probabilities using some of the simple survival models introduced in week 2 of MTH6157 Survival Models.

1. Simple force of mortality and survival probability calculations

If mortality follows the **Exponential Model** then force of mortality is constant μ and we can simply assign a value in R

mu = 0.00249

then ${}_tp_x = e^{-\mu t}$ for any x

so, if we set t = 5 say

tpx = exp(-mu*t)

tpx

returns [1] 0.9876272

If mortality follows **Makeham's Law** then $\mu_x = A + Bc^x$ for some A, B, c which in R can be calculated for example

A = 0.00006

B = 0.000002

c = 1.3

mu = function(x) { A + B * c ^ x }

mu(40)

returns [1] 0.002999543

we can then find ${}_tp_x$ by integration in R

tpx = function(x, t) {exp(-integrate(mu, x, x+t)$value) where $\int_o^t \mu_{x+s}$ ds is found by integrate(mu, x, x+t) in R and we need to add the $value command at the end otherwise R will return both the numeric value of the integral and a bound for the absolute error.

Here then ${}_{10}p_{40}$ is then found by

tpx(40, 10) which returns [1] 0.9191503

2. Exponential distribution calculations

If T_x ~ Exp(μ) some constant force of mortality independent of age x then there are some Exponential distribution functions built into R.

The pdf $f_T(t)$ is given by the R command

dexp(<t>, <mu>) for some given constant force of mortality assigned to mu

The Cumulative Distribution Function (which is in fact ${}_tq_x$ in survival models) is

```
pexp(<t>, <mu>)
```

and so the Survival Function (or ${}_tp_x$) is 1 - pexp(<t>, <mu>)

3. Gompertz Law

Under Gompertz Law the force of mortality is given by $\mu_x = Bc^x$ for some B, c.

In R the *flexsurv* package is useful for Gompertz Law calculations

```
install.package("flexsurv")
library(flexsurv)
```

to get started with this package. Rather than B, c constants R uses *shape* and *rate* parameters with Gompertz where

shape = ln c

rate = B

and an example of the set-up for this is

```
B = 0.000002
c = 1.3
```

`shape = log(c)`

`rate = B`

The force of mortality μ_x or **hazard** under this Gompertz Law can be calculated in R without using the B, c formula by `hgompertz(<x>, shape, rate)`

then `dgompertz(<t>, shape, rate)` returns the pdf $f_T(t)$ and certain durations t

and pgompertz(<t>, shape, rate) returns the Cumulative Distribution Function (or the ${}_tq_x$ probability) with `1 - pgompertz(<t>, shape, rate)` giving the Survival Function or ${}_tp_x$ probability at certain t.

The `rgompertz(<n>, shape, rate)` command simulates future lifetime values (T_x) from the Gompertz distribution with the given shape and rate, returning n values.