MTH6142 Complex Networks

Assessed Coursework 4

Note: This is an open book coursework, you are encouraged to use the propositions derived in class. (Typical solutions should fit in one or two pages). All submissions should be handwritten and in a .pdf format.

The reproduction number R of an epidemic spreading process taking place on a random network with degree distribution $P(k)$ is given by

$$R = \lambda \frac{\langle k(k-1) \rangle}{\langle k \rangle}, \tag{1}$$

where k indicates the degree of the nodes and the average $\langle \ldots \rangle$ indicates the average over the degree distribution, $P(k)$.
Therefore R is the product between the infectivity λ of the virus, due to its biological fitness and the branching ratio of the network, depending on the degree distribution of the network and given by $\langle k(k-1) \rangle / \langle k \rangle$.

According to the value of R the epidemic can be in different regimes:

- If $R > 1$ the epidemics is in the *supercritical regime*: the epidemics spreads on a finite fraction of the population, resulting in a pandemics.
- If $R < 1$ the epidemics is in the *subcritical regime*: the epidemics affects a infinitesimal fraction of the population and can be considered suppressed.
- If $R = 1$ the epidemics is in the *critical regime*: this is the regime that separates the previous two regimes.

Consider an epidemics with infectivity $\lambda = 1/4$. Investigate how the network topology can determine the regime of the epidemics in the following cases.

(A) Consider a Poisson network with average degree $c = 3$ and a Poisson network with average degree $c = 5$. Calculate R and establish in which regime the epidemic process is in these networks. **[1 MARK]**

(B) Calculate R for a scale-free network with degree distribution $P(k) = Ck^{-\gamma}$, minimum degree m, maximum degree K and power-law exponent $\gamma = 2.5$ using the continuous approximation for the degrees. **[2 MARKS]**

(C) Take the scale-free network considered in point (B) calculate R and establish in which regime the epidemic process is if $m = 2, K = 50$. **[1 MARK]**